Transition to the Property Tax in China: A Dynamic General Equilibrium Analysis

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Abstract

We use the dynamic general equilibrium approach to examine the potential impacts of the introduction of alternative tax regimes in China. This approach models housing demand and supply based on the dynamic optimization behaviors of households and firms. By choosing model parameters carefully, the model is able to capture key features in the Chinese real estate market, providing predictions about policy effects. We consider a universal property tax, a selective tax that applies to investment property only, and a land value tax. Regarding the use of tax revenue, we examine two alternatives: 1) a “redistributive” scenario where tax revenue is redistributed in equal amounts to households, and 2) a “non-redistributive” scenario where tax revenue is spent on local public goods but has no impact on housing decisions of households. Quantitatively we examine how different type and usage of tax revenue and expenditure scenarios impact land price, house price, house rent, vacancy rate and government revenue as well as land concession revenue.

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Introduction

Here we undertake a quantitative evaluation of the transition to a property tax system in China. The diversification of revenue sources will place urban finances on a more sustainable footing while fostering financial discipline among local governments. The goal of this paper is to explore alternative property tax and government spending regimes, and their implications for land concession revenue as well as (i) real estate prices, (ii) rents, (iii) land prices, (iv) revenue of local governments and (v) vacancy rate. Sub-national governments (cities) have depended to a large degree on sales of land use rights (land concessions), a revenue source which is expected to decline. In this paper we plan to employ a dynamic general equilibrium approach to explore the implications of the transition from the sale of land use rights to alternative sources of revenue.

The transition involves interesting dynamics. With the introduction of a property tax, house prices and land prices should fall due to the reduced return of housing investment. This would reduce government revenue from land concessions. Note that since local governments control the supply of new land for development new revenue sources would not replace land concession revenue but rather augment it. Presumably local governments would continue to supply land for development at market prices. Possible alternatives to solely depending on land concession revenue which are to be explored include: 1) a property tax plus land concession revenue; 2) a selective property tax targeting multiple house owners plus land concession revenue; and 3) a land value tax plus land concession revenue. It would be valuable to know whether this restructuring of land concession revenue can be compensated for by the new revenue from the alternative taxing regimes. It could be the case that total revenue is lowered initially, then reaches a higher level in the long run. A dynamic general equilibrium model is suitable for evaluating this complicated transition.

In the current environment, with continuing high appreciation rates in many of the first tier cities in China despite an overall decline in the GDP growth rate there is much discussion of the property tax as a mechanism to impose carrying costs on investors in multiple housing units or to simply moderate demand. There is a broader perspective which focuses on the creation of a sustainable urban fiscal system which has taken a back seat due to concerns about rapid price appreciation. In this paper we shed useful light on the impact on housing prices and rent as well as local government revenue.

Vacancy rate of residential houses in China has been under intense debate, despite the fact that no official statistics exists regarding the rate. It is widely speculated that some form of

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property tax should lower vacancy rate. In this paper we quantitatively evaluate the effects of various tax schemes on vacancy rate. Generally we find that tax policies are more effective in lowering vacancy rate if the tax revenue is transferred back to households than if the tax revenue is spent on public goods.

## Literature Review

The transition in Chinese real estate markets to a property tax system has been the subject of discussion for some time. Man (2011) provides an overview of local public finance in China and Anderson (2011) and Hong and Brubaker (2011) discuss the issues involved in transitioning to a property tax system from one where local public revenue has been derived in large part from the sale of land use rights (land concessions) in the form of long term leaseholds. Brys et al. (2013) include the recurring property tax as one among a broad range of potential changes in the structure of the tax system in China. There are multiple taxes currently levied on real estate summarized in the note below.

A deed tax of 3% of the purchase price is paid at the time of property purchase and has recently been reduced to 1 to 1.5% in most cities in order to revitalize the slowing residential real estate market with the exception of first-tier cities including Beijing, Shanghai and Shenzhen where real estate markets remain buoyant. A sales tax of 5.5% of the value of a property held for less than five years (previously two years) is levied on the seller. User fees could be another source of local revenue which allow the burden to be placed directly on the user of specific services such as waste collection and water consumption. For most cities in China, charges for urban services that can be linked directly to the user are not

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2They itemize property related taxes including 1) The “house property tax” is a recurrent tax on immovable property, mostly levied on the original purchase price or construction value net of 10%-30% of the value (provinces can choose the specific depreciation rate); the tax is not levied in rural areas. 2) Currently, China levies two land taxes. The “urban and township land use tax” is a recurrent (yearly) tax imposed on the land in cities, county towns, administrative towns and industrial and mining districts. The “arable (farming) land occupancy tax” is an area-based property transaction tax which has to be paid by the entities and individuals who build residential property on the arable land or who use the land for other non-agricultural purposes. 3) The “deed tax” is a property transaction tax which is levied on the (transaction) price of land and/or residential property and has to be paid by the transferee. 4) The “stamp tax” is a property transaction tax typically levied on securities and immovable property. As noted here there is an additional tax of 5.5% if a property is sold prior to five years. As well, all homeowners are subject to a capital gains tax of 20% but are exempt from that tax if the property is held for more than five years.
effectively used.\footnote{3}

Most developing economies view the strengthening of sub-national governments as an important policy agenda. \cite{BahlBird2008} note that in China a large portion of expenditure is decentralized but with virtually no independent taxing authority. With pressure to develop infrastructure this has led to extensive borrowing by local governments which has raised concerns about the level of indebtedness. Also, local governments have looked to the sale of land use rights as a revenue generator perhaps leading to more rapid development than justified by market conditions. Land concession revenue has been the prime source comprising about 70% of local revenue in 2010 declining to 45% in 2013.

In principal, both taxing and spending for local services should be more decentralized. \cite{Wong2013} concludes that the current situation for funding urban infrastructure is unsustainable and that China has to assign revenue and responsibilities such that incentives are better aligned in the decentralized, increasingly mobile society China has become. \cite{ChoiSjoquist2014} argue that the system of lump sum grants from Beijing produces distorted interests among suppliers and consumers of land and conclude that the property tax is a superior alternative source of revenue for local governments. The idea of devolving the responsibility for taxing and spending for certain public services to regional and local governments is discussed by \cite{Oates1998} and in most developed economies taxing and spending related to local infrastructure and services have evolved in this direction with a consequent focus on effectiveness of delivery.\footnote{3}

With the property tax a common source of local government revenue in developed economies, there is a significant theoretical and empirical literature. \cite{OatesSchwab2009} present a nice summary of the issues to be explored including the introduction of the tax and whether it is revenue neutral, the trade-off between a land tax and a property tax and the implications for the timing of development and sprawl.

\cite{George1958} and others have argued for a land value tax as such a tax is neutral with respect to development decisions and the burden falls totally on land owners. The result should be that land will more likely be developed when it is economic to do so. In an edited volume \cite{DyeEngland2009} summarize the theory and practice along with the evidence from empirical studies. Specifically, the international experience is summarized in \cite{Franzsen2009} and empirical study outcomes are summarized in \cite{Anderson2009}. \cite{OatesSchwab1996} summarize the theoretical research and report empirical results from the Pittsburgh,\footnote{3}

\footnote{3 The joint report by the World Bank and the Development Research Center of China’s State Council, has as one of six priorities for a new model of urbanization, “moving to a revenue system that would ensure a higher portion of local expenditures is financed by local revenues, such as property taxes and higher charges for urban services” \url{https://openknowledge.worldbank.org/handle/10986/18865}.}

\footnote{4 In developed economies this has spawned a literature on competition among municipalities based on the idea that consumers and firms shop with their feet. See \cite{Tiebout1956} and \cite{Breuckner2000}. In China, such mobility is restricted by Hukou.}
Pennsylvania experience with the land value tax. From the taxpayers perspective it is a wealth tax that places a greater burden on landowners and can be politically difficult to introduce. This combined with practical challenges including the higher volatility of land prices relative to property values and the difficulty of assessing the land component of the property have resulted in limited adoption despite the economic arguments in favor.

See Nitikin et al. (2012) for a discussion of the prospects for reform in China. There are both technical and political challenges with respect to the introduction of the property tax. From a technical perspective, the tax base needs to be defined and measured with the tax rate driven by local budget requirements and the existing plethora of taxes replaced by a broad based property tax levied at the local level. A not insignificant issue is defining the tax base in a leasehold environment where in theory the leasehold value drops as the remaining lease term falls. The incentives associated with this aspect of property rights in China are addressed in another context by Anglin et al. (2014). Ding (2005) sets out specific suggestions regarding changes to the existing system. Of note is that many proponents of the property tax in China focus more on their role in reducing high rates of price appreciation than on the longer term benefits of fiscal management at the local level.

While experiments with property taxes have been conducted in Shanghai and Chongqing, widespread change has not come as rapidly as initially planned or hoped. Observers suggest that the Shanghai and Chongqing experiments have not been successful for a number of reasons. First, each experiment was targeted; in Shanghai the tax was imposed only on newly purchased houses (excluding first purchases by local residents) and in Chongqing only on high end houses. Second, although tax rates varied from 0.4% to 1.2% homes with average values in Chongqing were taxed at 0.5% and in Shanghai at 0.4%. Moreover in Chongqing, value associated with the first 180 square meters of a single family house value was exempt and for other houses value associated with the first 100 square meters was exempt. So the effective tax rates were relatively low. Obviously in Chongqing homebuyers could avoid the tax by buying less expensive homes.

Using a simulation approach, Du and Zhang (2015) found that the trial property tax in Chongqing reduced the annual growth rate of housing prices by 2.52 percent and the trial property tax in Shanghai had no significant effect on housing prices. It is fair to say that these experiments provide little insight into the impact of broad imposition of a property tax. The taxes in the experiments are not realistic nor is there any examination of the benefits, if any, associated with the taxes.

According to the China Household Finance Survey 2011-2013, the vacancy rate of urban houses is 19.5% and about 21% of urban households own more than one housing unit. Housing units became a useful place to store wealth given low carrying costs, expected price appreciation and limited alternative investment opportunities. Another analysis in 2012 estimated that one

third of Chinese home owners owned two housing units\(^6\). The implementation of a property tax would alter a homeowner’s incentive to hold multiple housing units or to leave the houses vacant thus impacting the vacancy rate. The effects of various tax schemes are evaluated in our dynamic general equilibrium framework.

Some have argued that local governments have encouraged rapid development of urban land in order to enhance current local revenue shifting revenue from the future to the present. In the simulation exercise, we are able to explore the implications of alternative revenue generators. We consider different mixes of land concession revenue, the property tax, a selective tax on multiple home owners and the land value tax with the goal of identifying transition paths that are smoother and more sustainable.

We do not intend to address the political constraints regarding the transition to an alternative tax system and the decentralization of some taxing authority along with related spending, but rather focus on the likely short run and long run outcomes that would arise with various assumptions about the nature of the tax and its incidence as well as the nature of the benefits and their incidence. We also presume that over time that the tools to introduce, manage and monitor a decentralized property tax will be in place. Among these is the assessment or valuation of long term land leases (the mechanism for making land concessions or granting land use rights) in order to determine the tax base. This by itself is a significant challenge. We believe, however, that many homeowners in China do not think about the potential of lease renegotiation at the end of the initial lease term. For residential properties, the revised constitution allows for automatic extension at termination without addressing the terms of such extension.

There are at least four approaches available for studying the potential effects of a change in taxation regime on the housing market and land market: (i) the empirical approach; (ii) the static partial equilibrium approach; (iii) the computable general equilibrium approach (CGE); and (iv) the dynamic general equilibrium approach. We use a dynamic general equilibrium approach with the goal of examining the impact under various assumptions regarding the portfolio of taxing mechanisms and fees and the distribution of the benefits.

A reliable empirical analysis has at least two prerequisites. First of all, it requires sufficient data. For our research, the required data include housing price and land price before and after the changes of taxation regimes, as well as a set of covariates that also affect housing price and land price. The only incidences of property tax regime change in China are the property tax experiments in Shanghai and Chongqing launched in 2011 explored by Bai et al. (2014). Also see Du and Zhang (2015). With limited data and city specific policies, it is extremely difficult to conclude anything reliable regarding the actual impacts of a property tax. It is also difficult

\(^6\) Retrieved from https://chovanec.wordpress.com/2012/07/31/whats-driving-chinas-real-estate-rally-part-3. At that time, Chinese households owned 1.2 housing units with a home ownership rate of 88% (high by international standards).
to use them to infer the potential effects of property tax on other cities, given that different cities have tremendously different levels of economic development, patterns of migration, and land availability. Another reason why the empirical approach may not work well for the Chinese market is that the economy is in a transition process. Most likely the empirical relationships identified from the data may not hold true in the future, hence invalidating any predictions.

Using an alternative approach, Cao and Hu (2016) conduct a micro-simulation using a static partial equilibrium approach. They use the China Family Panel Survey (CFPS) data to show that a uniform property tax policy would bring heterogeneous impacts across different income groups as well as different regions, mainly due to the differences in income distribution, housing prices and the degree of the Housing Demolition program. The latter influence arises from compensation for demolition of houses when new construction takes place. As will be explored in this paper they find that redistribution of the tax revenue can reduce the regressivity of the tax. They use cross-sectional information in the Chinese Family Panel Survey (CFPS) data to simulate optimal tax scenarios for each region. However, this approach while providing insight into the varied impact of the tax in a heterogeneous market, does not generate long run impacts on the market equilibrium.

Several previous studies have employed the CGE approach. See Choi and Sjoquist (2014), Andrew (2004) and Nechyba (1998) for studies focused on introducing a land value tax and Julia-Wise et al. (2014) for a study focused on a statewide property tax limitation. These CGE models are static and cannot take into consideration the dynamic effects of tax regime changes. For example, the implementation of a property tax may cause households to switch from renting to owning later in life. So longer term impacts on tax revenue cannot be captured by a static CGE model.\footnote{To predict the tax effect, an empirical or partial equilibrium model must rely on the elasticity of the supply of and demand for the taxed property. A changing tax regime will certainly change the elasticities, thereby invalidating any prediction.}

While there are challenges with the empirical or CGE model alternatives, the dynamic general equilibrium approach meets our needs well. This approach models housing demand and supply based on the dynamic optimization behavior of households and firms. By choosing model parameters carefully, the model is able to capture the key features in the Chinese housing market and provide predictions about policy effects. These model parameters are related to the household’s preferences and the firm’s production technology; hence they are immune to the Lucas Critique. See Lucas (1976).
Background to the Modeling Approach

There are numerous hypotheses to be addressed all to do with the timing and impact of the following revenue sources (Table 1) on real estate rents and prices, the vacancy rate and government revenue. We consider a system with four sources of revenue for local governments: 1) land concession revenue (the sale of land use rights); 2) a property tax which is proportional to property value; 3) a selective property tax proportional to property value imposed on multiple home owners; and 4) a land value tax proportional to land value. As noted previously, we presume that in all cases the local government continues to derive revenue from land concessions.

We examine two alternative expenditure scenarios: 1) a “redistributive” strategy where tax revenue is transferred equally to all so that low income households are benefited more proportionally; and 2) a “non-redistributive” strategy where the tax revenue is spent on local public goods so that each household receives the same proportional increase in the utility level, but it does not impact housing investment or rental choice. The goal is to explore implications of the transition for (1) real estate prices, (2) rents, (3) land prices, (4) the revenue and budget of local governments and (5) the vacancy rate.

<table>
<thead>
<tr>
<th>Table 1: Sources of Government Revenue</th>
</tr>
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<tbody>
<tr>
<td>Redistributive</td>
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<tr>
<td>----------------</td>
</tr>
<tr>
<td>Land concession income (sale of land use rights)</td>
</tr>
<tr>
<td>Universal property tax</td>
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<tr>
<td>Selective property tax</td>
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<tr>
<td>Land value tax</td>
</tr>
</tbody>
</table>

This table summarize the eight case scenarios we analyze in this paper. The numbers in the parentheses label the case scenarios to be studied.

We adopt the model developed in Han et al. (2017). The model has a number of distinct features:

(i) It allows for non-stationarity in the growth rates of income, population, land supply, house price, rent and land price. The non-stationarity of variables poses a challenge to both empirical and theoretical analysis. To solve this problem, our model constructs a balanced

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8 It is perhaps a trivial point but we presume local governments would continue to sell land concessions for their market value. The alternatives would be at a below market price effectively subsidizing the buyer or, alternatively, perhaps providing more compensation for rural collectives. The latter could be part of a broader redistributive strategy. We leave that possibility for later consideration.
growth path (BGP) to which the housing market will converge after the Chinese economy becomes a developed one. From the BGP, we solve for the equilibrium paths for the transition period of the economy using backward induction.

(ii) The model endogenously generates the market clearing paths of house price, rent and land price. To study the effects of taxation regime change, it is necessary for these prices to be endogenous. By our specially designed algorithm, we are able to compute the whole paths of endogenous prices, both before and after the policy change.

(iii) The model is by nature quantitative. It is able to explain the recently observed price and rent in the Beijing housing market. We achieved this by calibrating the model parameters carefully according to the Chinese data. For example, we use micro-level income panel data from the China Health and Nutrition Survey to estimate the stochastic income process for the Chinese households. We also use the age-specific fertility rate, mortality rate and the age distribution of population to predict the evolution of population in the future, which is important in light of Mankiw and Weil (1989).

(iv) The model takes into account the high but declining income growth in China. This is important. When asked about the Chinese housing market, Robert Shiller the Nobel Prize Laureate told the media: “China is in such a rapid growth period. It is very hard to price assets when growth is at the high level. The future matters more. In a stable economy that is not going anywhere, you have a pretty good idea of what they are worth”.

(v) The model is able to generate a high savings rate in China which is partly responsible for the high housing price as studied by Wang and Wen (2012). The high savings rate in China has become a “puzzle”. See Ma and Yang (2013). In our model, household saving is driven by three channels. The first channel is precautionary saving. Our model allows for random shocks to individual income and allows households to use housing investment to buffer against income shocks. The second channel is stochastic medical expenses which are shown by Chamon and Prasad (2010) to be one of the main reasons for Chinese households to save. The third channel is the bequest motive. By choosing parameter values in the bequest function properly, the model is able to generate a strong bequest motive and hence high level of wealth for old age Chinese households. This strong bequest incentive is consistent with the observation that many homes of young families in China are purchased with the help from parents.

Both a practical and modeling issue has to do with determining property value as it is influenced by the terms of the land concession or land use rights granted. Most residential property in China is on land leased for seventy years (Anglin et al. 2014). While most property on leased land declines in value over the term of the lease as shown by Dale-Johnson (2001), the situation in China is unclear. According to the Property Law of the People’s Republic of China passed in 2007, Article 149 states “The term of the right to the use of land for building houses shall automatically be renewed upon expiration. The term of the right to the use of
land for non-house building purpose shall be renewed according to laws and regulations upon expiration. With regard to ownership of the houses built on the land and other real property related, the relevant agreement (if any) shall be abided by, or, if there is no such agreement, the relevant provisions stipulated by law and administrative regulations shall be observed.\footnote{Retrieved from \url{http://faolex.fao.org/docs/pdf/chn72735.pdf}.

Notably, while extension is automatic for houses or owned apartments, there is no specification of the terms of the extension. Some early developments were on shorter leases and recently a potential sale of a Wenzhou condominium illustrated the issue. The sale of a home with an underlying 20 year lease close to expiry triggered a demand for a payment by the seller to the local government of $46,500 (US) as if the land use right was being resold for a new lease term of 70 years. Here we do not address this issue, we presume that leases will automatically extend at no cost.\footnote{A few month after the Wenzhou event, the Ministry of Land and Resources published an official correspondence stating that when the land use rights reach expiration it should be automatic extended without a fee (\url{http://www.mlr.gov.cn/zwgk/zytz/201612/t20161223_1425017.htm}). This is the most recent interpretation of the Property Law. The issue of extension terms is not separate from the imposition of a property tax. One way to ensure sustainable property tax revenue is to allow for automatic extension of the lease at no cost except in the event of redevelopment.}

A further concern is vacancy. In China, as noted earlier many urban households own more than one housing unit which often remain vacant so it is important to consider the impact of various tax scenarios on housing investment. To study the vacancy rate, we assume in the beginning of each period a home owner receives a random shock called ”renting cost”. This represents the costs associated with renting houses to tenants including search, information, transaction and maintenance costs as well as moral hazard. If the home owner draws a renting cost that is higher than the rent, then she leaves the house vacant. Different tax regimes in our model lead to different changes in rent, which in turn endogenously generate a different vacancy rate. Vacancy lowers the return on housing investment thereby lowering housing demand, and through the general equilibrium effect, housing supply will also be lowered by vacancy.

In summary, note that option (2) and (3) in Table 1 both involve an ad valorem property tax, an annual property tax which is a percentage of property value, a tax which is widely used in developed economies. Option (4) is a land value tax. In options (2) and (4), all home owners are taxed based on the property value or land value respectively. Option (3) is a selective property tax where only multiple home owners are taxed on an ad valorem basis. The presumption is that local jurisdictions will continue to generate revenue from the sale of land concessions but that revenue will be influenced by the price and volume of house purchases and investments.

The effects of the transition from land-based public finance to a new system clearly depend on how tax revenue is spent. We consider two distinct types of expenditure. The first alternative is “redistributive” where the tax revenue is equally transferred to all households so that low
income households are benefited more proportionally. The second alternative is where the tax revenue is spent locally but does not impact housing investment or rental choice. For example, the expenditure would be made on local infrastructure. All households benefit but not through housing investment or consumption. We refer to this as “non-redistributive” because expenditures evenly benefit all households in the form of a multiplier to the utility level.  

Model

In the economy there exist overlapping generations of households and a long-lived representative firm. The households receive exogenous income which is directly consumable, and purchase housing services from the firm. The firm uses land and capital to build houses. We assume land supply is exogenously determined by the government that sells some new land to the market in each period. But land price is endogenously determined so that the land market clears. To finance the land purchase and capital investment, the firm issues shares. Following Kiyotaki et al. (2011), we assume households are shareholders of the firm. Each household optimally decides whether to own or rent houses.

There exists no aggregate uncertainty in this economy. Households face a common deterministic growth rate of income, although they experience idiosyncratic income shocks and medical expense shocks. There is also no productivity shock. Consequently house price is non-stochastic.  

Firm

The representative firm maximizes the shareholder value. It combines land and capital to produce houses. The production function and dynamic optimization problem are laid out below.

Production Function

Letting $K$ and $L$ denote capital and land input, the firm’s production function is

$$H_t = Z L_t^\theta K_t^{1-\theta},$$

(1)

11 The non-redistributive tax would still involve redistribution if expenditure is not equally valued by the rich and poor.

12 This should lead to higher housing demand and higher house price relative to the case of risky housing investment.
where $Z$ is a scaling parameter, and $\theta \in (0, 1)$ measures the relative importance of land in housing construction. As in Kiyotaki et al. (2011), our specification assumes that the firm can continuously adjust housing production.\(^{13}\)

We abstract away from labor input in housing construction for simplicity and transparency. In an extended version of the model (available upon request), we include labor input in the model and find its impact on equilibrium house price for Beijing to be minor. This is consistent with the empirical finding in the previous studies that land price is much more important than labor cost in determining the house price for big cities.\(^{14}\)

### Timing and Flow of Funds

At the start of period $t$, the firm already owns $H_{t-1}$ units of housing produced using $K_{t-1}$ units of capital and $L_{t-1}$ units of land. Without loss of generality, we normalize the number of shares in housing firm equity to be the same as the number of housing units. At the beginning of period $t$, denote per unit house price (also price per share of housing equity) as $p_{t-1}$ and per unit rent (also dividend per share) as $r_t$. The firm collects rental income $r_t H_{t-1}$ and pays it out to shareholders as dividends. Then the firm issues new shares to raise capital and purchase land for the construction of new housing. At the end of period $t$ (after the issuance of new shares), the number of total housing units becomes $H_t$, and the house price becomes $p_t$.

The firm’s flow of funds in period $t$ is

$$p_t(H_t - H_{t-1}) = K_t - (1 - \delta)K_{t-1} + q_t(L_t - L_{t-1}),$$

where $\delta$ is the depreciation rate of capital, and $q_t$ is land price in period $t$. The left side of the equation above represents the proceeds from issuing new shares, which are used to purchase additional capital and land, as shown on the right hand side of (2).

### Optimization Problem

At the beginning of period $t$, the firm decides on the purchase of new capital and land to maximize value for existing shareholders after the issuance of new shares, $p_t H_{t-1}$. From equation (2),

\(^{13}\)Here we adopt the zero adjustment cost assumption, which is common in models that build upon the neoclassical growth theory. Under this assumption, our production function is identical to an alternative specification where the developer firm produces new housing units each period with newly supplied land and capital. In reality, downward adjustment of housing stock is difficult, at least in the short run. However, we focus in this paper on a growing economy with no productivity shocks, hence downward adjustment never happens.

\(^{14}\)See Davis and Heathcote (2007) for evidence on the US market and Wu et al. (2012) for evidence on the Chinese market.
we have
\[ p_t H_{t-1} = p_t H_t - [K_t - (1 - \delta)K_{t-1}] - q_t (L_t - L_{t-1}) \\
= \frac{r_{t+1} + p_{t+1}}{(r_{t+1} + p_{t+1})/p_t} H_t - [K_t - (1 - \delta)K_{t-1}] - q_t (L_t - L_{t-1}) \\
= \frac{1}{R_{t+1}} (r_{t+1} H_t + p_{t+1}H_t) - [K_t - (1 - \delta)K_{t-1}] - q_t (L_t - L_{t-1}) \\
= \frac{r_{t+1}}{R_{t+1}} H_t + \frac{1}{R_{t+1}} p_{t+1}H_t - [K_t - (1 - \delta)K_{t-1}] - q_t (L_t - L_{t-1}) \\
= \tilde{r}_t H_t - [K_t - (1 - \delta)K_{t-1}] - q_t (L_t - L_{t-1}) + \frac{1}{R_{t+1}} p_{t+1}H_t \\
\]
where \( R_{t+1} \) is the total return on housing equity between \( t \) and \( t + 1 \), defined as
\[ R_{t+1} = \frac{r_{t+1} + p_{t+1}}{p_t}. \]

Note that \( R_{t+1} - 1 \) is also the cost of financing for the firm. We define \( \tilde{r}_t = \frac{r_{t+1}}{R_{t+1}} \), so that \( \tilde{r}_t H_t \) is the present value at \( t \) of rental income to be collected at the beginning of next period.

Therefore, the firm’s value and optimal decisions regarding the new stocks of capital and land depend on the stocks of capital and land carried over from the previous period, denoted \( K_{t-1} \) and \( L_{t-1} \) respectively. In the dynamic programming problem of the firm, the state vector is \((K_{t-1}, L_{t-1})\). Using \( V(K_{t-1}, L_{t-1}) \) to denote the value of the firm given the state vector, the firm’s optimization is
\[
\max_{K_t, L_t} \tilde{r}_t H_t - [K_t - (1 - \delta)K_{t-1}] - q_t (L_t - L_{t-1}) + \frac{1}{R_{t+1}} V(K_t, L_t) \\
\text{s.t.} \quad H_t = ZK_t^{1-\theta}L_t^\theta. \tag{5}
\]

First order conditions with respect to \( K_t \) and \( L_t \) are
\[
Z(1 - \theta) \tilde{r}_t \left( \frac{K_t}{L_t} \right)^{-\theta} = 1 - \frac{1}{R_{t+1}} \frac{\partial V(K_t, L_t)}{\partial K_t} = 1 - \frac{1 - \delta}{R_{t+1}} \tag{6}
\]
\[
Z\theta \tilde{r}_t \left( \frac{K_t}{L_t} \right)^{1-\theta} = q_t - \frac{1}{R_{t+1}} \frac{\partial V(K_t, L_t)}{\partial L_t} = q_t - \frac{q_{t+1}}{R_{t+1}} \tag{7}
\]

In the last equality of (6) and (7), we have used the envelope conditions: \( \frac{\partial V(K_t, L_t)}{\partial K_t} = 1 - \delta \) and \( \frac{\partial V(K_t, L_t)}{\partial L_t} = q_{t+1} \).

From equation (6), capital input relative to land, \( \frac{K_t}{L_t} \), decreases with \( R_{t+1} \), because higher \( R_{t+1} \) means a higher cost of capital. On the other hand, equation (7) shows that \( \frac{K_t}{L_t} \) increases with the cost of land. The firm can acquire land at the price of \( q_t \) in period \( t \), and then sell it for \( q_{t+1} \) in the next period. Discounting land price in period \( t + 1 \) to period \( t \) using \( R_{t+1} \), the cost of one unit of land is thus \( q_t - q_{t+1}/R_{t+1} \). Intuitively, higher land price induces substitution of capital for land, leading to a higher \( \frac{K_t}{L_t} \).
Optimal Housing Supply

In this subsection, we first derive the optimal housing supply as a function of the exogenous land supply in period $t$ denoted by $L^*_t$. Then we determine the market clearing land price by adjusting the land price so that land demand by the firm equals the exogenous land supply.

The optimal level of capital given land supply $L^*_t$, house rent $r_t$ and financing cost $R_{t+1}$ can be obtained from the first order condition (6):

$$K^*_t = \left[ \frac{Z(1-\theta)\tilde{r}_t}{1-(1-\delta)/R_{t+1}} \right]^{1/\theta} L^*_t \quad (8)$$

Plugging this expression into the housing production equation (1), we derive the following housing supply function:

$$H_t = Z^{1/\theta} \left[ \frac{(1-\theta)\tilde{r}_t}{1-(1-\delta)/R_{t+1}} \right]^{(1-\theta)/\theta} L^*_t \quad (9)$$

Thus in our model, housing supply depends critically on land supply. When more land is supplied by the government, the firm optimally chooses more capital investment as shown in equation (8), and hence housing supply is increased.

From equation (9), everything else being equal, housing supply decreases with housing equity return (which is also the cost of financing for the housing firm) and increases with the rent. Thus, when there is excessive investment demand, price $p_t$ increases and $R_{t+1}$ falls, so that supply rises; when housing consumption demand exceeds housing supply, rent $r_t$ increases so that supply rises. This helps us design the algorithm that searches numerically for the market clearing paths of house price and rent.

Market Clearing Land Price

Using the land market equilibrium condition that the firm uses $L^*_t$ unit of land, equation (7) becomes

$$\theta\tilde{r}_tH_t = \left( q_t - \frac{q_{t+1}}{R_{t+1}} \right) L^*_t. \quad (10)$$

Recall that $\theta$ is the land share in the Cobb-Douglas housing production function, and $\tilde{r}_tH_t$ is the firm’s revenue in period $t$. The left side of the equation (10) is the share of revenue attributed to land as one of the production factors, while the right side is the cost of land.

Equation (10) can be rewritten into the following relation which can be used to solve land price recursively:

$$q_t = \frac{\theta\tilde{r}_tH_t}{L^*_t} + \frac{q_{t+1}}{R_{t+1}} \quad (11)$$

Intuitively, current land price $q_t$ equals the land share of the firm’s revenue (per unit of land), plus the discounted future land price. We will show that the economy has a BGP in which
land price grows at a constant factor of $G_q$, and the firms financing cost is a constant, denoted $R_{BGP}$. Therefore in the BGP, land price can be expressed as a function of $G_q$, $R_{BGP}$, and rent.

To solve for the land price in the BGP, we obtain the following dynamic relation for land prices from equations (9), (10), and (11)

$$q_t = \frac{q_{t+1}}{R_{t+1}} = \theta Z^{1/\theta} \tilde{r}_t^{1/\theta} \left[ \frac{1 - \theta}{1 - (1 - \delta)/R_{t+1}} \right]^{(1-\theta)/\theta}$$

In the BGP, $q_{t+1} = q_t G_q$ and $R_{t+1} = R_{BGP}$, thus

$$q_t = M \frac{\tilde{r}_t^{1/\theta}}{1 - G_q^{1/\theta}/R_{BGP}},$$

where $M = \theta Z^{1/\theta} \left[ \frac{1 - \theta}{1 - (1 - \delta)/R_{BGP}} \right]^{(1-\theta)/\theta}$ is a function of $R_{BGP}$, a constant denoting the firm’s financing cost in the BGP.

In the Proposition we will show that rent $r_t$ grows at a constant factor $G_r$ in the BGP. Moreover, $G_q = G_r^{1/\theta}$. Plugging this into equation (13), we have:

$$q_t = M \frac{\tilde{r}_t^{1/\theta}}{1 - G^{1/\theta}/R_{BGP}},$$

where $M = \theta Z^{1/\theta} \left[ \frac{1 - \theta}{1 - (1 - \delta)/R_{BGP}} \right]^{(1-\theta)/\theta}$ is a function of $R_{BGP}$, a constant denoting the firm’s financing cost in the BGP.

That is, in the BGP, land price $q_t$ is the sum of discounted rental rates from period $t$ on raised to the power of $1/\theta$.

If we denote the housing rent and the land price respectively by $r_{BGP}$ and $q_{BGP}$ at the time the economy reaches the BGP, then

$$q_{BGP} = M \frac{r_{BGP}^{1/\theta}}{1 - G^{1/\theta}/R_{BGP}}.$$ 

In the quantitative analysis that follows, we first derive $q_{BGP}$ from $r_{BGP}$ and $R_{BGP}$ which in turn are obtained from a set of regularity conditions. Then we compute equilibrium paths of house price and rent via backward induction, and back out the land price path during the economic transition based on equation (12). This is the major step in solving our model numerically.
Households

The economy is populated by a growing mass of households. A household works since beginning at age $J_0$ and retires at $J_1$, then lives up to a maximum age of $J$. At each age, a household is faced with an age-specific death probability. In the numerical analysis, $J_0 = 21$, $J_1 = 60$ and $J = 96$. We choose ages which reflect a realistic life cycle.

Households choose between owning and renting, therefore home ownership is an endogenous outcome in the model. They also choose housing and non-housing consumptions as well as investments to maximize life-time utility. Homeowners can invest in both the risk-free asset and housing equity, but the only vehicle of saving for the renters is the risk-free asset.

Utility function and bequest value

We assume the Cobb-Douglas utility for households in each period:

$$u(c, h) = \frac{(c^{1-\omega}h^{\omega})^{1-\gamma}}{1-\gamma},$$

(16)

where $h$ is the housing consumption (the size of house one lives in, either rented or owned), $c$ is the non-housing consumption, $\omega$ is a parameter measuring the relative importance of housing consumption in utility, and $\gamma$ is the inverse of intertemporal elasticity of substitution (EIS). This utility form implies a unit elasticity of substitution between housing and non-housing consumption for which Morris and Ortalo-Magne (2011) find strong data support. Further, it implies that in the BGP of our model, consumption, investment, house price and rent grow at constant rates.

At the end of period $t$, a household of age $a$ dies with probability $\nu_a$. We assume that the value of bequeathing $s_a$ shares of equity at age $a$ is

$$V_b(s_a) = \max_{c, h} B\ u(c, h),$$

s.t.

$$c + r_th = p_ts_a,$$

where parameter $B$ determines the strength of the bequest motive. In other words, the deceased evaluates the utility as if the beneficiaries consume the bequeathed wealth in just one period, optimally splitting it between housing and non-housing consumption. With the Cobb-Douglas preference, the bequest value of equity shares when a homeowner dies is:

$$V_b(s_a) = B \left[ (1-\omega)^{1-\omega} \omega \right]^{1-\gamma} \left( \frac{1}{r_t} \right)^{\omega(1-\gamma)} \left( p_ts_a \right)^{1-\gamma} \frac{1}{1-\gamma}.$$

(17)

See the Section entitled “Evolution of Population Structure” for details. Households face mortality risks that increase with age. Assets of the households that leave the economy are distributed evenly among the young households aged $J_0$. 

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Similar arguments show that the bequest value of $b_a$ amount of risk-free asset when a renter dies is:

$$V_b(b_a) = B \left[ (1 - \omega)^{1-\omega} \omega^\omega \right]^{1-\gamma} \left( \frac{1}{r_t} \right)^{\omega(1-\gamma)} \frac{(b_a)^{1-\gamma}}{1-\gamma}. \quad (18)$$

**Heterogeneity and Uncertainty**

The households are ex ante homogeneous, but they are heterogeneous ex post because they receive idiosyncratic income shocks and medical expense shocks. In addition, the households face mortality risks which increases with age as in the data. As mentioned earlier, we assume there is no aggregate uncertainty in the economy.

We include out-of-pocket medical expense shocks in the model for two reasons. First, recent studies show that stochastic medical expense is an important determinant of wealth accumulation/decumulation for retirees (see [De Nardi et al. 2010](#) and the references therein). Second, medical insurance is usually under-provided in the emerging markets, and our framework can serve as a tool to examine the potential impact of better provision of medical insurance. The model could, of course, be revised to consider other expense shocks but ready data allows the consideration of medical expenses.

The model admits two major incentives of wealth accumulation: precautionary savings and retirement savings. The vehicle of saving is equity shares of the firm.

**Income and Medical Expense**

Income growth is one of the key driving forces of housing demand. Household income consists of two components, one deterministic and the other stochastic. Let $y(i, a, t)$ be the income of the $i^{th}$ household at age $a \leq J_1$ and year $t$, then

$$y(i, a, t) = \tilde{y}(i, a, t) \times \overline{y}(a, t), \forall a \leq J_1, \quad (19)$$

where $\tilde{y}(i, a, t)$ and $\overline{y}(a, t)$ are the stochastic and the deterministic components respectively. The deterministic income, $\overline{y}(a, t)$, includes an age effect ($a$) capturing the hump-shaped lifecycle profile of income and a time effect ($t$) for the growth of the aggregate income. We assume an AR(1) process for the logarithm of stochastic component of income:

$$\ln \tilde{y}(i, a, t) = \rho_y \ln \tilde{y}(i, a - 1, t - 1) + \epsilon(i, a, t), \forall a \leq J_1, \quad (20)$$

where $\epsilon(i, a, t)$ is the idiosyncratic shock to the $i^{th}$ household in year $t$, and $\rho_y$ determines the persistence of the shock. Regardless of time and the household’s age, $\epsilon(i, a, t)$ is drawn from a normal distribution with mean zero and standard deviation of $\sigma_y$. For a household just entering the labor market, the age is $J_0$ and we assume $\tilde{y}(i, J_0, t) = \epsilon(i, J_0, t)$. 

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After retirement, households are no longer subject to income shocks. At the time of retirement, income of a household is assumed to be $\kappa$ fraction of its income right before retirement. Further, post-retirement income is assumed to grow at the same rate as the aggregate income.

Retirees are faced with stochastic out-of-pocket medical expenses, denoted by $m(i, a, t)$, which is assumed to have an idiosyncratic stochastic component $\tilde{m}(i, a, t)$ and a deterministic component $m(a, t)$ that is common for all individuals of the same age $a$ at time $t$:

$$m(i, a, t) = \tilde{m}(i, a, t) \times m(a, t), \forall a > J_1.$$  

(21)

We assume that $\ln \tilde{m}(i, a, t)$ follows an AR(1) process:

$$\ln \tilde{m}(i, a, t) = \rho_m \ln \tilde{m}(i, a - 1, t - 1) + \eta(i, a, t), \forall a > J_1,$$

(22)

where $\eta(i, a, t)$ is drawn from a normal distribution with mean zero and standard deviation $\sigma_m$. Since out-of-pocket medical expense is small relative to income before retirement, we assume it is zero for simplicity.

**Housing Investment and Leverage**

A household enters the economy as a renter and saves via the risk-free asset before becoming a homeowner. Barriers to homeownership include the minimum down payment requirement and the minimum (starter) housing size. To buy a house of size $s$ at price $p$, a household needs to pay down at least $d \times p \times s$ where $d$ is the minimum down payment, and $s \geq \underline{s}$, the minimum housing size. Thus a household needs to save at least $d \times p \times \underline{s}$ before she becomes a homeowner.

Homeowners can use mortgages to borrow up to $1 - d$ of the house values. Thus, they hold leveraged investments in housing equity. The mortgage interest rate $r_m$ is exogenous. There is no mortgage default in our model. Given the housing fundamentals in the model, the equilibrium housing return is certain and positive. Households that experience negative idiosyncratic income or medical expense shocks can choose smaller housing investments or exit the housing market completely. We assume mortgage adjustment, such as refinancing, is costless.

Suppose a household of age $a$ holds $s_{a-1,t-1}$ shares of housing equity at the end of $t - 1$. At the beginning of the period $t$, the household’s non-financial income $y_{a,t}$ and out-of-pocket medical expense $m_{a,t}$ are revealed. Housing firm dividend per share $r_t$ is also realized and paid to the existing shareholders, and then housing equity is traded. The household decides whether to enter the housing market and becomes a homeowner if $s_{a-1,t-1} = 0$, or change the holding of housing equity if $s_{a-1,t-1} > 0$. In some cases, the homeowner may find it optimal to exit the housing market completely and becomes a renter. For example, old households tend to have an incentive to exit the housing market despite the high housing return. This is because
their non-housing consumption may be sub-optimal if they keep their housing assets above the minimize house size (which becomes too big for old households).

If the household decides to enter or stay in the housing market, she needs to decide on the quantity of housing consumption $h_{a,t}$, nonhousing consumption $c_{a,t}$ and shares of equity $s_{a,t}$. If the household decides to exit or stay out of the housing market, she needs to decide on the amount of risk-free asset holding $b_{a,t}$, as well as $h_{a,t}$ and $c_{a,t}$.

In our model, both housing and risk-free asset are safe and it is costless to adjust housing investment. Consequently it is always optimal to invest only in the asset that provides the higher return. Since house price and rent are endogenously determined in our model, given an exogenous risk-free rate, house price and rent adjust until the leveraged return on the housing investment is sufficiently high so that the demand for housing equity equals the supply of it in each period. In other words, the leveraged return on the housing asset exceeds the return on the risk-free asset in each period. Therefore, homeowners invest in housing asset only, and hold zero risk-free asset unless they exit the housing market when their income is too low to maintain the minimum housing size. Renters save in the form of the risk-free asset. After they have saved enough for the minimum down payment, they can become homeowners and invest in the housing equity.

**Household’s Optimization Problem**

For ease of presentation, we omit the household index $i$ and the time index $t$ from the choice variables of households, but keep the age index $a$. For aggregate variables such as price and rent, the time index is still used.

**The homeowner’s problem** For a homeowner of age $a$ who enters the current period $t$ with $s_{a-1}$ shares of housing equity, income $y_a$ and medical expense $m_a$, we use $V^{rent}(s_{a-1}, y_a, m_a)$ to denote her value if she exits the housing market and becomes a renter this period, and $V^{own}(s_{a-1}, y_a, m_a)$ to denote the value if she stays in the housing market. The value function of a homeowner given the state vector $(s_{a-1}, y_a, m_a)$ is

$$V(s_{a-1}, y_a, m_a) = \max \{V^{rent}(s_{a-1}, y_a, m_a), V^{own}(s_{a-1}, y_a, m_a)\}.$$ 

Specifically,

$$V^{own}(s_{a-1}, y_a, m_a) = \max_{s_a, h_a} u(c_a, h_a) + \beta \mathbb{E} \left[ (1 - \nu_a) V(s_{a+1}, y_{a+1}, m_{a+1}) + \nu_a V_b(s_a) \right],$$ 

s.t. 

$$r_t h_a + c_a = y_a - m_a + [p_t + r_t - LTV_t - p_t(1 + r_m)]s_{a-1} - (1 - LTV_t) p_t s_a,$$

and $s_a \geq s_a$. 

where in the budget constraint, $\nu_a$ is the probability of death by the end of the period for someone of age $a$, $V_b(s_a)$ is the bequest value of equity shares given in (17), and $LTV_t$ is the
loan-to-value ratio of the homeowner in period $t$ given by

$$LTV_t \begin{cases} 
1 - d, & \text{if } r_m < R_{t+1}; \\
0, & \text{if } r_m \geq R_{t+1};
\end{cases}$$

where $r_m$ is the mortgage rate, and $R_{t+1}$ is the total return on the housing equity defined in equation (4). That is, homeowners use leverage via the mortgage market and borrow up to the limit (i.e., they only pay the minimum down payment) if the mortgage rate is lower than the housing equity return, and borrow nothing otherwise. The right hand side of the budget constraint for the homeowner takes into consideration the interest payment of the outstanding loan balance as well as the change in the loan size as the household adjusts the equity holding. Since the homeowner obtains a leveraged position in housing equity using mortgage, the actual rate of return $r_{lev,t+1}^{s,t}$ she receives from housing investment between $t$ and $t + 1$ is given by

$$r_{lev,t+1}^{s,t} = \frac{1}{1 - LTV_t} (R_{t+1} - 1) - \frac{LTV_t}{1 - LTV_t} r_m.$$ 

In a similar vein, the value function of a household that enters the current period as a homeowner but switches to a renter this period is given by:

$$V_{rent}(s_{a-1}, y_a, m_a) = \max_{b,a,h_a} u(c_a, h_a) + \beta \mathbb{E} \left[ (1 - \nu_a) W(b_a, y_{a+1}, m_{a+1}) + \nu_a V_{b}(b_a) \right],$$

subject to $r_t h_a + c_a = y_a - m_a + \left[ p_t + r_t - LTV_{t-1} p_{t-1} (1 + r_m) \right] s_{a-1} - b_a,$

and $b_a > 0,$

where $W(b_a, y_{a+1}, m_{a+1}),$ to be defined below, is the next period’s value function of a renter given the state vector $(b_a, y_{a+1}, m_{a+1}).$

**The renter’s problem** Consider a renter household of age $a$ who enters the current period $t$ with $b_{a-1}$ amount of risk-free asset. In the beginning of the current period, her income $y_a$ and out-of-pocket medical expense $m_a$ are realized. Let $W_{rent}(b_{a-1}, y_a, m_a)$ denote her value function if she continues to rent in the current period, and $W_{own}(b_{a-1}, y_a, m_a)$ denote her value function if she enters the housing market this period. The household compares the value of renting vs. owning to decide whether to enter the housing market. Overall, the value function of a renter given the state vector $(b_{a-1}, y_a, m_a)$ is

$$W(b_{a-1}, y_a, m_a) = \max \{ W_{rent}(b_{a-1}, y_a, m_a), W_{own}(b_{a-1}, y_a, m_a) \}.$$  

Specifically,

$$W_{rent}(b_{a-1}, y_a, m_a) = \max_{b,a,h_a} u(c_a, h_a) + \beta \mathbb{E} \left[ (1 - \nu_a) W(b_a, y_{a+1}, m_{a+1}) + \nu_a V_{b}(b_a) \right],$$

subject to $r_t h_a + c_a = y_a - m_a + (1 + r_b) b_{a-1} - b_a,$

and $b_a > 0.$
where $V_b(b_a)$ is the bequest value of risk-free investment given in (18), and $r_b$ is the exogenous risk-free rate.

$$W_{own}(b_{a-1}, y_a, m_a) = \max_{s_a, h_a} u(c_a, h_a) + \beta \mathcal{E} \left[(1 - \nu_a)V(s_a, y_{a+1}, m_{a+1}) + \nu_a V_b(s_a)\right],$$

s.t. 
$$r_t h_a + c_a = y_a - m_a + (1 + r_b) b_{a-1} - (1 - LTV_t) p_t s_a,$$

and $s_a \geq \bar{s}$.

where $V(s_a, y_{a+1}, m_{a+1})$ is the next period’s value function of a homeowner defined above.

### Optimality Conditions

In each period, a household makes both intra-temporal and inter-temporal decisions. Inter-temporally, the household’s optimal consumption-saving choices satisfy

$$\frac{\partial u(c_a, h_{a,t})}{\partial c_a} = \beta \mathcal{R}_{t+1} \mathcal{E} \left[(1 - \nu_a) \frac{\partial u(c_{a+1}, h_{a+1})}{\partial c_{a+1}} + \nu_a V'_b(w_a)\right],$$

(23)

where $w_a = s_a$, $\mathcal{R}_{t+1} = 1 + r_{lev}^{w_a}$ for the homeowners who invest in the housing market and $w_a = b_a$, $\mathcal{R}_{t+1} = 1 + r_b$ for the renters who save with the risk-free asset. In the equation above, we have used the envelope condition that the derivative of a household’s value function with respect to asset holding equals the marginal utility of consumption when evaluated at the optimal choices.

Intra-temporally, the decision is to allocate between housing and non-housing consumption. At the optimal choices, the marginal utility of housing consumption and the marginal utility of non-housing consumption satisfy the following condition:

$$\frac{\partial u(c_a, h_{a,t})}{\partial h_a} = r_t \frac{\partial u(c_a, h_{a,t})}{\partial c_a}.$$ 

(24)

Under the Cobb-Douglas utility function $u(c, h)$ in (16), this is equivalent to:

$$\frac{c_a}{h_a} = r_t \frac{1 - \omega}{\omega}.$$ 

(25)

Equations (23) and (25), together with the household’s budget constraint, determine the amount of housing and non-housing consumptions in current period and the amount of housing asset that carries over into the next period.

### General Equilibrium

The general equilibrium is defined as sequences of house prices $p_t$, rents $r_t$, land prices $q_t$, and sequences of choices made by the firm and the households that satisfy the following conditions:

(i) the firm’s choices of $L_t$, $K_t$, $H_t$ are consistent with the firm’s optimization problem; (ii)
the household’s choices of consumptions (housing and non-housing) and investments (risk-free asset and housing equity) are consistent with the household’s optimization problem; (iii) the distribution of households in the state space evolves according to the law of motion specified below; and (iv) the paths of prices and rents satisfy market clearing conditions specified below.

A number of objects concepts need to be defined precisely before we specify the market clearing conditions and the law of motion. Households are heterogeneous in the holding of risk-free asset, holding of housing asset, age, income and medical expense, as represented by the state vector \((b, s, a, y, m)\). In each period, households are distributed in the five dimensional state space of non-negative real numbers, denoted \(B \times S \times A \times Y \times M\), with \(b \in B\), \(s \in S\), \(a \in A\), \(y \in Y\), \(m \in M\).

Let \(\lambda_t(b, s, a, y, m)\) be the distribution of households in the state space at the beginning of period \(t\). Let \(B \times S \times A \times Y \times M\) be a typical subset of the state space. The probability that households with current state \((b, s, a, y, m)\) transition into the set \(B \times S \times A \times Y \times M\) at the end of period \(t\) is

\[
Q_t(B \times S \times A \times Y \times M|b, s, a, y, m) = \int_{(a', y', m') \in A \times Y \times M} I\{(b', s') \in B \times S|b, s, a, y, m\} \Gamma(a', y', m'|a, y, m)da'dy'dm',
\]

where \(I\) is the indicator function, and \(b'\) and \(s'\) are the optimal decision rules regarding risk-free bond and housing equity investment solved from the household’s optimization problems given the state of \((b, s, a, y, m)\). Recall that the survival probability, the transition of stochastic income and the transition of stochastic medical expenses over time are all exogenous. Therefore we use \(\Gamma(a', y', m'|a, y, m)\) to denote the exogenous probability of the transition from state \((a, y, m)\) to state \((a', y', m')\), where \(a' = a + 1\) if the household survives.

**Law of Motion of the Distribution of Households**

For the local residents, the law of motion of the distribution of households in the whole state space (the product of the exogenous space \(A \times Y \times M\) and the endogenous space \(B \times S\)) is as follows:

\[
\lambda_{t+1}(B \times S \times A \times Y \times M) = \int_{(b, s, a, y, m) \in B \times S \times A \times Y \times M} Q_t(B \times S \times A \times Y \times M|b, s, a, y, m)d\lambda_t(b, s, a, y, m)
\]

For migrants of each age cohort, we assume that their distribution in the space \(B \times S \times Y \times M\) is identical to the distribution of local residents of the same age, and the distribution of these

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\(^{16}\)This is a general state vector for both renters and owners. In the earlier discussion of the homeowners and renters problems we have introduced state vectors for homeowners and renters individually conditional on their age.
local residents evolves according to equation (26). In the beginning of the next period, the
migrants are included as part of the local residents.

Market Clearing Conditions

In period $t$, the housing consumption market clears if:

$$H_t = \int_{(b,s,a,y,m) \in B \times S \times A \times Y \times M} h'_{t}(b, s, a, y, m) d\lambda_{t}(b, s, a, y, m)$$  \(27\)

where $h'_{t}(b, s, a, y, m)$ is the optimal housing consumption given the state $(b, s, a, y, m)$.

The equity market clears if:

$$H_t = \int_{(b,s,a,y,m) \in B \times S \times A \times Y \times M} s'_{t}(b, s, a, y, m) d\lambda_{t}(b, s, a, y, m)$$  \(28\)

where $s'_{t}(b, s, a, y, m)$ is the optimal housing investment given the state $(b, s, a, y, m)$. The
aggregate outcome of all housing investment demand must equal the supply which is denoted by $H_t$.

Finally, the land market clearing condition is:

$$L_t = L_t^*$$

where $L_t$ is the firm’s land demand and $L_t^*$ is the exogenous land supply in period $t$.

Balanced Growth Path

Assume that from year $T_{BGP}$ forward, aggregate income, land supply and population grow at
fixed factors $G_Y$, $G_L$ and $G_N$ respectively. In addition, age distribution of population no longer
changes over time. Then we have the following proposition (with proof provided in Appendix A) about the properties of the BGP.

**Proposition.** A BGP exists and is characterized by the following:

1. Aggregate capital growing at a factor of $G_K = G_Y$;
2. Aggregate housing supply growing at a factor of $G_H = G_Y^{1-\theta} G_L^\theta$;
3. Housing investment demand and consumption per capita growing at $G_s = (G_Y/G_N)^{1-\theta} (G_L/G_N)^\theta$ and $G_h = G_s$;
4. Demand for risk-free asset per capita growing at a factor of $G_Y$;
5. Non-housing consumption per capita growing at $G_c = G_Y/G_N$;
6. House price growing at $G_p = (G_Y/G_L)^\theta$;
7. House rent growing at $G_r = (G_Y/G_L)^\theta$;

8. Land price growing at $G_q = G_Y/G_L$;

9. Floor-area ratio, defined as $H/L$, growing at $G_{FAR} = (G_Y/G_L)^{1-\theta}$;

10. Constant price-income ratio ($pH/Y$) and price-rent ratio ($p/r$).

Equilibrium in the BGP has a set of properties that are consistent with stylized facts. For example, house price is driven by income growth and land supply. The importance of income is shown in Case and Shiller (2003), and the importance of land supply is emphasized in Glaeser and Saks (2005) and Saiz (2010). It has been observed that house price indices, after controlling for inflation, can exhibit extremely low growth rates in the long run. This phenomenon can be generated in our model when land supply and aggregate income grow at similar rates. On the other hand, when land supply grows at a lower rate than income, the model predicts a growing trend of house price, consistent with the pattern of house price experienced in the past decades by cities such as Hong Kong, San Francisco and New York.

By comparing the sixth and the eighth points in the Proposition, we can see that the growth rate of land price is always higher than that of house price since $\theta < 1$. This is consistent with the empirical observations of major cities in both the U.S. in Davis and Heathcote (2007) and China in Deng et al. (2012). In addition, price-income ratio and price-rent ratio are both constants in the BGP. This is consistent with Ambrose et al. (2013) who document that price-rent ratio exhibits long-run stationarity.

It should be noted that the economy does not operate in the BGP immediately after the stabilization of the exogenous variables. It needs to wait until the age distribution and the asset distribution of households become time-invariant. In the quantitative analysis below, we assume that after year 2044, all exogenous variables grow at constant rates. After another 70 years, i.e., after 2114, the age distribution and asset distribution will be time-invariant. Therefore, the Beijing housing market enters into BGP in 2114 under our model.

The key variables grow at constant rates in the BGP, therefore they can be re-scaled so the economy operates as if it is in a steady state. In the quantitative analysis, we start with finding the house price and rent in this “steady state”; then we use them as the terminal conditions to solve for the equilibrium paths of price and rent during the transition periods using backward induction.

\[17\text{See, e.g., Chart 4 in Shiller (2007).}\]
Tax Policies

Property Tax on All Home Owners

We assume that every house is taxed. Further, we assume that a property tax rate of $\tau$ is implemented at the end of year 2017. This tax is not expected, thus households make their housing decision according to zero property tax prior to the end of 2017. With the announcement of some $\tau > 0$, household re-optimize which changes their demand for housing investment and housing consumption. Consequently each of house price, house rent and land price adjust as a consequence of the imposition of the tax.

Regarding the use of government revenue from property tax, we consider two cases. First, the revenue is not re-distributed back to households but is spent on local infrastructure. Second, it is re-distributed evenly to each household. In the first case, the budget constraint of households is now the following

$$r_t h_a + c_a = y_a - m_a + p_l(s_{a-1} - s_a) + r_t s_a - \tau p_l s_a$$

i.e., property tax is a fraction of the value of property. Clearly, the tax tightens the budget constraint. A key point of this study is to understand how this affects prices and government revenue in general equilibrium.

In the second case, let $tr_t$ be the transfer from the government that comes from property tax at time $t$, the budget constraint is

$$r_t h_a + c_a = tr_t + y_a - m_a + p_l(s_{a-1} - s_a) + r_t s_a - \tau p_l s_a.$$  

Selective Property Tax on Owners of Multiple Homes

Only owners who invest in multiple housing units are taxed. The Shanghai pilot property tax program essentially selectively taxes home owners who own multiple properties. Using our model, we can also study the quantitative effect of this tax policy. In this case, there is a switch $M \in \{0, 1\}$ for equation (29)- (30) such that property tax rate is zero when $M = 0$, and tax rate is $\tau$ when $M = 1$. The definition of $M$ is the following.

$$M \begin{cases} 
= 0, & \text{if } s_a \leq h_a; \\
= 1, & \text{if } s_a > h_a.
\end{cases}$$

To be clear, owners of multiple homes are defined as those whose housing investment exceeds housing consumption.
Land Value Tax

Another alternative is to impose a land value tax, as suggested by Henry George. Again let $\tau$ be the tax rate. This tax will affect the optimal decision of the housing home developer (the firm). The budget constraint of the firm as shown in equation (2) is re-written as

$$p_t(H_t - H_{t-1}) = K_t - (1 - \delta)K_{t-1} + q_t((1 + \tau)L_t - L_{t-1}), \quad (31)$$

The value function of the firm, i.e., equation (5) also needs to be changed accordingly into

$$\max_{K_t, L_t} \tilde{r}_t H_t - [K_t - (1 - \delta)K_{t-1}] - q_t((1 + \tau)L_t - L_{t-1}) + \frac{1}{R_{t+1}}V(K_t, L_t)$$

$$s.t. \quad H_t = ZK_t^{1-\theta}L_t^\theta. \quad (32)$$

The first-order condition with respect to land is now

$$Z\tilde{r}_t \left( \frac{K_t}{L_t} \right)^{1-\theta} = (1 + \tau)q_t - \frac{q_{t+1}}{R_{t+1}}. \quad (33)$$

Notice that the right-hand of equation (33) is the marginal cost of land acquisition. Given tax rate $\tau$, this marginal cost is apparently higher, which affects the optimal allocation of land and capital, favoring more use of capital given the same land price. But we will show that land price will be lowered by this land value tax.

The housing supply function (9) is derived from the housing production function and first-order condition with respect to capital, and therefore is not affected by a land value tax. This may appear counter-intuitive. However it is a natural result of the assumption that land supply is exogenously determined by the government. Given the same land supply, the land value tax lowers land price so that land demand is unchanged by the tax. Consequently, land input is the same as before the imposition of the land value tax regime.

The equation governing land price dynamics as in equation (11) is re-written into

$$q_t = \frac{1}{1 + \tau} \left( \frac{\tilde{r}_t H_t}{L_t} + \frac{q_{t+1}}{R_{t+1}} \right). \quad (34)$$

Similarly, land price in the BGP is changed into

$$q_t = M \frac{\tilde{r}_t^{1/\theta}}{1 + \tau - \frac{G_q}{R_{BGP}}}. \quad (35)$$

From equation (34)-(35), it’s obvious that land price is reduced by the land value tax during each time period. Hence the tax will reduce government income from land concessions.

For each property tax scenario, we also consider two cases regarding the alternative uses of revenue from the new taxation regime – non-redistributive and redistributive. In the former case, neither households decisions nor housing supply is affected by the land value tax.
Consequently, both house price and house rent remain unchanged. In the latter redistributive case, with the redistribution of tax revenue to households, all households have more to spend generating stronger demand for housing, hence higher house prices and rents. The impact of each tax regime and the interplay with the benefit assumptions will be discussed in more detail in the Results section.

Projection and Calibration

Having studied the analytical features of the housing market in the BGP, we now turn to the transition path where the non-stationarity of prices and fundamental variables become important. Like most of the incomplete market models in which idiosyncratic income and medical expense shocks are non-insurable, the model does not admit an analytical solution for the transition path. Therefore we resort to numerical methods. In the context of the Beijing market, this section explains in detail the projection of exogenous processes, the estimation of initial conditions, and the calibration of model parameters.

Projections of Population, Income, and Land Supply

Evolution of Population Structure

Two dimensions of the population structure are relevant for our model: population size and the age distribution of population. Population size directly affects housing demand. The age distribution matters because housing demand is age-specific. Both housing consumption demand and housing investment demand have hump-shaped age profiles. Therefore an aging population generates lower aggregate housing investment demand.

The population data are obtained from the 2010 National Census, and from sample surveys in other years between 2005-2013. The upper-left panel of Figure 1 shows the age distribution of Beijing residents, defined as individuals who either have formal registration (Hu Kou) or have lived in Beijing for more than half a year. Compared with the overall urban population, the Beijing population is much younger due to the influx of young in-migrants.

To project the population structure after 2013, we need to predict the fertility rate, mortality rate and immigration rate of the Beijing population. Using the data between 2005-2013, we

---

18 The 2005 sample is 1% of Beijing population, associated with the nationwide 1% Population Sample Survey. Sample size is about 0.1% between 2006-2009 which are associated with nationwide Sample Survey on Population Change. Since 2011, Beijing Bureau of Statistics has routinized annual sample survey to be 2% of the population in non-census years.
calculate the age-specific fertility rate and mortality rate of the Beijing population, shown in the upper-right (Old Rate) and lower-left panel of Figure 1. The low fertility rate in the data is due to China’s one-child policy (which is currently being relaxed) and implies ever decreasing total population if we assume it remains the same in the future. Thus, we consider an alternative specification for the age-specific fertility rate (see the line labeled “New Rate” in the upper-right panel of Figure 1): for each age and in the first 10 years starting from 2014, it is the same as that estimated using the data between 2005-2013, but it rises linearly for the next 10 years so that the overall population growth rate reaches 0.4% in 2034 and then fertility rate is assumed to be time-invariant afterwards. Mortality rate is assumed to be constant over time since life expectancy in China is already close to those in industrialized countries.

As emphasized in [Henderson (2010)](http://data.stats.gov.cn/workspace/index?m=csnd), rapid urbanization is one of the key issues in population dynamics for a developing country. Urbanization is reflected in the increasing city population in our model. We define immigration rate as the number of new immigrants to Beijing as a fraction of the existing Beijing population. The rate has been declining since 2008, and averages around 2.88% between 2010-2013. In the baseline model, we assume that immigration rate decreases linearly from 2.88% in 2014 to zero after 30 years. In the data it is clear that immigrants to Beijing are mainly young workers, therefore we assume that only those aged between 20-30 migrate to Beijing in each year.

Based on the 2013 data and using the fertility rate, mortality rate and immigration rate discussed above, we extrapolate the population structure after 2013. The lower-right panel of Figure 1 plots the projected age structure of population in 2020, 2060 and 2100. Upon the completion of urbanization which is represented by a zero immigration rate after 2044, the peak age of population moves to 65 in 2060. In year 2100, the population structure stabilizes to a profile that decreases with age, due to the increasing age-profile of mortality rate.

**Evolution of Land Supply**

In China, local governments own land and auction land use rights (land concessions). The amount of land to be auctioned depends on a multitude of considerations, including policies from the central government, fiscal balance of the local governments and the growth rate of local GDP.

We obtain data on the supply of residential land between 2005 and 2013 from the National Bureau of Statistics (NBS), and project the land supply onward. For each major city in China, NBS reports the amount of new residential land acquired by housing developers. This is the flow of land. The stock of land in 2009 is available from the 2010 *China Statistical Year Book of Environment* complied by NBS. Table 11-3 of the year book is “Basic Statistics on Urban Area and Land Used for Construction by Region” which reports that the area of residential

[^19]: http://data.stats.gov.cn/workspace/index?m=csnd
Figure 1: Fertility rate, mortality rate, and population structure

This figure shows the initial age distribution of population, fertility rate, mortality rate and projected population structure. The fertility rate is presented as one half of women’s fertility rate in the data, interpreted as fertility rate per couple. The “Old Rate” is the average fertility rate between 2005-2013 in the data. The “New Rate” is the projected fertility rate after 2034.

Land is 383.3 hectares in Beijing at the end of 2009. Based on the stock of land in 2009 and the annual flows, we obtain the total stock of residential land in Beijing. Finally, we divide total land stock by the population of Beijing residents to obtain land supply per capita.

Table 2 reports total residential land, newly-acquired residential land, and residential land per capita. Although the simulation of the economy starts from 2005, we report land supply since 2001. From the last row of the table, it is clear that the growth rate of land falls far behind that of population after 2005, leading to declining land supply per capita, which is partly responsible for the soaring house prices since 2005 and reflected by the increasing number of high-rise residential buildings.

We assume that land supply grows at a constant rate of 0.05% from 2014 onwards in the baseline model. The evolution of land per capita is given by the solid line in the left panel of Figure 2. Land supply per capita falls gradually due to the influx of population as urbanization continues. As the growth of population plateaus, land supply per capita becomes time-invariant. The two broken lines show the evolution of land per capita when the growth of aggregate land supply is either 1% or 0%, which will be used in the sensitivity analysis.
Table 2: Land and population of urban Beijing

<table>
<thead>
<tr>
<th></th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population (million)</td>
<td>13.9</td>
<td>14.2</td>
<td>14.6</td>
<td>14.9</td>
<td>15.4</td>
<td>16.0</td>
<td>16.8</td>
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<tr>
<td>Growth rate (%)</td>
<td>2.75</td>
<td>2.33</td>
<td>2.49</td>
<td>3.03</td>
<td>4.10</td>
<td>4.68</td>
<td></td>
</tr>
<tr>
<td>New land (ha)</td>
<td>1472</td>
<td>2093</td>
<td>1391</td>
<td>1572</td>
<td>774</td>
<td>295</td>
<td>392</td>
</tr>
<tr>
<td>Total land stock (ha)</td>
<td>29518</td>
<td>30990</td>
<td>33082</td>
<td>34474</td>
<td>36046</td>
<td>36820</td>
<td>37115</td>
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<tr>
<td>Growth rate (%)</td>
<td>4.99</td>
<td>6.75</td>
<td>4.21</td>
<td>4.56</td>
<td>2.15</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>Land per capita (m²)</td>
<td>21.31</td>
<td>21.77</td>
<td>22.72</td>
<td>23.09</td>
<td>23.44</td>
<td>23.00</td>
<td>22.14</td>
</tr>
<tr>
<td>Growth rate (%)</td>
<td>2.18</td>
<td>4.32</td>
<td>1.67</td>
<td>1.48</td>
<td>-1.87</td>
<td>-3.71</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population (million)</td>
<td>17.7</td>
<td>18.6</td>
<td>19.6</td>
<td>20.2</td>
<td>20.7</td>
<td>21.4</td>
</tr>
<tr>
<td>Growth rate (%)</td>
<td>5.67</td>
<td>5.03</td>
<td>5.48</td>
<td>2.89</td>
<td>2.51</td>
<td>3.61</td>
</tr>
<tr>
<td>New land (ha)</td>
<td>823</td>
<td>625</td>
<td>859</td>
<td>507</td>
<td>306</td>
<td>906</td>
</tr>
<tr>
<td>Total land stock (ha)</td>
<td>37507</td>
<td>38330</td>
<td>38955</td>
<td>39814</td>
<td>40321</td>
<td>40627</td>
</tr>
<tr>
<td>Growth rate (%)</td>
<td>1.05</td>
<td>2.20</td>
<td>1.63</td>
<td>2.20</td>
<td>1.27</td>
<td>0.76</td>
</tr>
<tr>
<td>Land per capita (m²)</td>
<td>21.18</td>
<td>20.61</td>
<td>19.86</td>
<td>19.72</td>
<td>19.49</td>
<td>18.95</td>
</tr>
<tr>
<td>Growth rate (%)</td>
<td>-4.37</td>
<td>-2.69</td>
<td>-3.65</td>
<td>-0.67</td>
<td>-1.21</td>
<td>-2.75</td>
</tr>
</tbody>
</table>

Evolution of Aggregate Income

The average disposable income of Beijing residents, as reported by the NBS, is 36.47, 39.30 and 41.96 thousand Renminbi (RMB) in the years 2012, 2013 and 2014 respectively, each in terms of 2012 RMB. These numbers are roughly one-third of the disposable income numbers of Hong Kong residents in the corresponding years. We assume that average income grows at a constant rate of 7% in 2015, then the growth rate declines linearly from 7% to 3% over the next 30 years. Given the assumption that population grows at a constant rate of 0.4% in the BGP, the growth rate of average income is \((1 + 3%)/(1 + 0.4%) - 1 = 2.59\%\) per year. In the sensitivity analysis, we consider two alternative cases where income growth plateaus at 3% after either 20 or 40 years. Figure 2 plots the projected evolution of the per capita income under the baseline and the two alternative scenarios.

Initial Assets

The initial distribution of households’ assets by age is an important input in the model. The best survey data about household assets in China is the China Household Finance Survey (CHFS [http://www.chfsdata.org/](http://www.chfsdata.org/)). However, thus far there is only one wave of data available publicly – the 2012 wave. From the survey, we estimate the ratio of financial wealth to income and the age profiles of financial assets and housing equity for urban households in China. The estimated ratio of financial wealth to income is 3.68.

Data from the Beijing Bureau of Statistics show that the disposable income per capita is 19.13 thousand in 2005 (in terms of 2012 RMB). Therefore the estimated average financial wealth is 70.37 thousand RMB for the Beijing residents in 2005. In addition, the average housing size for Beijing residents is 19.5 square meters in 2005. We distribute these assets across different
This figure shows the projected land supply and income under different assumptions. In the left panel, "growth" refers to the growth rate of land per capita. In the right panel, “growth” refers to number of years it takes before the growth rate of aggregate income plateaus. In the baseline mode we use “growth = 0.5%” and “growth = 30 yrs”

Income and Medical Expense

The age profile of income, defined as the term $\bar{y}(a, t)$ in equation (19), is estimated from China Health and Nutrition Survey (http://www.cpc.unc.edu/projects/china/). We use all the available waves of survey prior to 2011 (i.e. 1989, 1991, 1993, 1997, 2000, 2004, 2006, 2009), and regress the logarithm of income on age and year dummies. The left panel of Figure 3 plots the smoothed age-profile of income, re-scaled to match the average income in 2014. In addition to the age profile of income, our numerical analysis takes into account the growth trend of income over time.

The age-specific medical expense as a proportion of income is estimated from the 2011 wave of China Health and Retirement Longitudinal Study (http://charls.ccer.edu.cn/en). As shown in the right panel of Figure 3, this ratio is 0.15 at age 61, and it reaches 0.45 at age 96.

The AR(1) processes of stochastic income and out-of-pocket medical expense are estimated from China Health and Nutrition Survey and China Health and Retirement Longitudinal Study.

\footnote{In the model, financial wealth is also treated as housing equity, therefore the initial housing stock in the model would be higher than in the data. To address that, in the simulation we distribute the 70.37 thousand RMB financial wealth to households who enter the economy in 2005 over a period of 10 years, between 2005-2014.}
respectively. The persistence parameters and variances of shocks are reported in Table 3.

<table>
<thead>
<tr>
<th></th>
<th>var. of shocks</th>
<th>persistence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income</td>
<td>0.064</td>
<td>0.864</td>
</tr>
<tr>
<td>Medical expense</td>
<td>0.25</td>
<td>0.922</td>
</tr>
</tbody>
</table>

Model Parameters

Our model has three parameters related to housing production \((Z, \theta, \delta)\) and four preference parameters \((\gamma, B, \beta, \omega)\) respectively. These parameters are pinned down by calibrating our model to match some key features of the Beijing market when it reaches the BGP. We assume that the Beijing housing market in the BGP will resemble the current state of Washington, D.C. in terms of price-income ratio, price-rent ratio and growth rate of real house price.

To pin down the four parameters related to consumer preferences, \((\gamma, B, \beta, \omega)\), we take a moment-matching approach. Specifically, we pick the set of parameters so that the following six moments generated from the model match as closely as possible those from the data: (i) average price-income ratio; (ii) average price-rent ratio; (iii) home ownership rate; (iv) the average age of first-time home buyers; (v) clearing of the rental market in the BGP (equation (27)); (vi) clearing of the housing equity market in the BGP (equation (28)).

To generate model moments, we simulate 1,000 paths of income and medical expenses for each generation of households, compute the optimal decisions of households in the BGP for each path, then calculate the related moments by taking the average values across the 1,000 simulated households.

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In a prior version of the paper, we used Hong Kong as a benchmark for the BGP. We now use Washington, D.C., because it is a city with a large government sector and relatively elastic land supply perhaps reflecting the nature of Beijing as the Chinese economy matures.
For each of the moments used in model calibration, Table 4 shows its target value and the fitted value from our calibrated model. The average age of first-time home buyers contains important information about preference for home ownership as well as parameters related to wealth accumulation, such as $\beta$ and $\gamma$. In the U.S., age at first marriage is 28 on average and age of first-time home purchase is 34 according to the 2009 American Housing Survey.

<table>
<thead>
<tr>
<th>Moments</th>
<th>Target</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>price/income</td>
<td>31.01</td>
<td>32.05</td>
</tr>
<tr>
<td>price/rent</td>
<td>9.08</td>
<td>9.06</td>
</tr>
<tr>
<td>home ownership rate</td>
<td>0.75</td>
<td>0.77</td>
</tr>
<tr>
<td>age of first time buyers</td>
<td>33</td>
<td>34</td>
</tr>
<tr>
<td>surplus (consumption MKT)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>surplus (investment MKT)</td>
<td>0</td>
<td>0.02</td>
</tr>
</tbody>
</table>

The home ownership rate target is 0.75 for the Beijing market in the BGP. According to data from the 2012 wave of the China Household Finance Survey, the average home ownership rate in the first-tier cities in China is over 80%. Based on the history of economies that have experienced successful economic transition, we do not expect the home ownership rate in Beijing to decline significantly in the future.

The price-income ratio and price-rent ratio of 31.1 and 9.08 respectively, calculated as the post-2010 average ratios observed in Washington, D.C. The resulting parameters are $\gamma=2.09$, $\beta=16.32$, $\beta=0.998$, and $\omega=0.25$.

For housing production parameters, it is evident from the Proposition that, in the BGP, the growth factors of house price ($G_p$) and land price ($G_q$) satisfy $G_p = G_q^\theta$, therefore

$$\theta = \frac{\log G_p}{\log G_q} \quad (36)$$

Thus we first pin down the land share parameter $\theta$ using equation (36) based on the growth rates of house price and land price in Washington, D.C. This gives $\theta = 0.60$.

To pin down the production efficiency parameter $Z$, we use equation (9) which implies

$$Z = FAR^\theta / \left[ \frac{1-(1-\theta)r}{1-(1-\delta)}R \right]$$

where $FAR$, the floor-area ratio in the BGP, is taken to be 3.


23 This number is lower than the present FAR in Hong Kong, but higher than the present FAR in Washington, D.C. Housing in Washington, D.C. is mainly stand-alone single-family houses while in Beijing, it is predominately condominiums. FAR=3 in the BGP is a more realistic assumption for Beijing because the FAR of condominiums built in recent years in Beijing already exceeds 2.5.
Using $r = 16.53$ USD per square foot and $R = G_p + (r/p) = 1.05$, we obtain $Z = 0.90$.

Finally, we infer the growth of land supply ($G_L$) from the sixth point in the **Proposition**, $G_p = (G_Y/G_L)^\theta$. For Washington, D.C., we obtain $G_Y = 1.056$ and $G_p = 1.020$, which yields $G_L = \exp(\log(G_Y) - 1/\theta \log(G_p)) = 1.02$. This is about the average land supply growth of seven U.S. cities featuring rich land supply: Chicago, Cincinnati, Los Angeles, Minneapolis, Philadelphia, Pittsburgh and Tacoma.\textsuperscript{24}

An additional input parameter in the model is the down payment rate ($d$). For the Beijing market, down payment is generally 30% for the first home and about 50-60% for the second home. We take the average and set $d = 0.4$. This is consistent with the average down payment rate of the middle income households in first-tier cities in China, as calculated in Fang et al. (2015).

**House Price, Rent and Land Price in the BGP**

We have used information from the Washington, D.C. market such as per-capita income, rental rate, land use, price-income and price-rent ratios to identify the scaling parameter $Z$. Now we use the projected per capita income and land in Beijing to obtain house price and rental rate when the Beijing market converges to the BGP.

From the housing supply equation (9), it is straightforward to obtain

$$
r_t = \left( \frac{1}{Z} \right) \left( \frac{\text{ratio}^{py}}{\text{ratio}^{pr}} \times \frac{Y_t}{L_t} \right) \theta \left[ \frac{1 - (1 - \delta)/R_{t+1}}{1 - \theta} \right]^{1-\theta} \tag{37}
$$

To calculate the rental rate when the economy enters the BGP, we need to predict $Y_{BGP}$ and $L_{BGP}$, the income and land supply in Beijing at $t = T_{BGP}$.

Based on the projected evolution of income, land supply and urban population structure, by the time the Beijing market reaches the BGP, $Y_{BGP} = 832$ thousand in 2012 RMB, and $L_{BGP} = 12.83$ square meters per capita. Substituting these numbers and other related values into equation (37), we obtain $r_{BGP} = 4.12$ thousand RMB per square meter. This is the market clearing annual rental rate for Beijing $t = T_{BGP}$ (i.e., year 2114).

House price at the BGP is calculated as $r_{BGP}$ divided by the price-rent ratio. That is $P_{BGP} = 4.12 \times 35.6 \approx 146.7$ thousand in 2012 RMB per square meters. Land price at the BGP is 816.0 per square meter, calculated using equation (15). Thus land price per square meter is 5.6 times that of house price per square meter at the time the economy converges to the BGP.

Table 5 reports the set of preference parameters with which the model best fits the data moments best. Our model is able to generate a high price-income ratio as well as other moments for the Beijing housing market in the BGP, if households are very patient (high $\beta$), have a high EIS (low $\gamma$) and have a very strong bequest motive (high $B$).

\textsuperscript{24}See \url{http://www.lincolninst.edu/subcenters/atlas-urban-expansion/global-sample-cities}. 

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\textsuperscript{33}
Table 5: Parameters

<table>
<thead>
<tr>
<th>Production parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>land share in production</td>
<td>$\theta$</td>
</tr>
<tr>
<td>capital depreciation rate</td>
<td>$\delta$</td>
</tr>
<tr>
<td>scaling parameter in production</td>
<td>$Z$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Preference parameters</th>
<th></th>
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<tbody>
<tr>
<td>inverse of EIS</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>discount factor</td>
<td>$\beta$</td>
</tr>
<tr>
<td>housing share in utility</td>
<td>$\omega$</td>
</tr>
<tr>
<td>strength of bequest motive</td>
<td>$B$</td>
</tr>
</tbody>
</table>

Results

This section reports the main quantitative results. We report the consequences of imposing a property tax in non-redistributive and redistributive scenarios at three different rates: three different types of tax of 0.5%, 1% and 2%. We report the impact on house price, house rent and land price for 1) a property tax on all owners, 2) a property tax on owners with multiple homes and 3) a land value tax.

Changes in the BGP

We first study how the tax policy changes house price, house rent and land price in the BGP. Analytically, growth rates of the key variables, as proposed earlier, remain unchanged in the BGP. But the tax changes the levels of these variables, because both consumption demand and investment demand of housing are altered by the tax policy.

Recall that the clearing of housing consumption and investment markets are obtained by choosing preference parameters of households. Imposing a property tax impacts housing demand so markets are no longer in equilibrium unless house price and rent are adjusted. We search for $r_{BGP}$ and $P_{BGP}$ such that both consumption and investment markets of housing are again in equilibrium. From the new $r_{BGP}$ and $P_{BGP}$, the new $q_{BGP}$ is also obtained.

The effects of a universal property tax on house price, house rent and land price when the economy enters the BGP are reported in the first panel of Table 6. Three patterns are evident. First, the tax has a negative effect on house price. Such negative tax effect is stronger when tax...
Table 6: Prices in the BGP

<table>
<thead>
<tr>
<th>tax rate ( (\tau) )</th>
<th>Non-redistributive</th>
<th>Redistributive</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>house price</td>
<td>house rent</td>
</tr>
<tr>
<td>0.5%</td>
<td>131.05</td>
<td>4.48</td>
</tr>
<tr>
<td></td>
<td>-8.7</td>
<td>-2.8</td>
</tr>
<tr>
<td>1%</td>
<td>119.85</td>
<td>4.45</td>
</tr>
<tr>
<td></td>
<td>-16.5</td>
<td>-3.6</td>
</tr>
<tr>
<td>2%</td>
<td>103.34</td>
<td>4.33</td>
</tr>
<tr>
<td></td>
<td>-28.0</td>
<td>-6.0</td>
</tr>
</tbody>
</table>

Property Tax (All Owners)

|                      | house price | house rent | land price | house price | house rent | land price |
| 0.5%                 | 131.17      | 4.49      | 471.67     | 130.92      | 4.91      | 497.59     |
|                      | -8.6        | -2.7      | -10.3      | -8.8        | 6.5       | -5.4       |
| 1%                   | 119.86      | 4.45      | 426.90     | 123.63      | 5.19      | 483.43     |
|                      | -16.5       | -3.6      | -18.8      | -13.8       | 12.5      | -8.1       |
| 2%                   | 103.61      | 4.35      | 358.85     | 115.10      | 5.28      | 451.41     |
|                      | -27.8       | -5.7      | -31.8      | -19.8       | 14.5      | -14.2      |

Property Tax (Owners With Multiple Houses)

|                      | house price | house rent | land price | house price | house rent | land price |
| 0.5%                 | 143.47      | 4.61      | 466.20     | 131.27      | 4.68      | 433.02     |
|                      | 0.0         | 0.0       | -11.3      | -8.5        | 1.4       | -17.7      |
| 1%                   | 143.47      | 4.61      | 418.70     | 120.60      | 4.74      | 365.75     |
|                      | 0.0         | 0.0       | -20.4      | -15.9       | 2.7       | -30.4      |
| 2%                   | 143.47      | 4.61      | 347.83     | 106.44      | 4.71      | 277.36     |
|                      | 0.0         | 0.0       | -33.9      | -25.8       | 2.2       | -47.3      |

Land Value Tax

This table reports house price, house rent, land price when the economy enters the BGP given different types of tax policy and different tax rates. The italic numbers are percentage changes in prices relative to the zero-tax regime.
rate is higher, or when the tax revenue is not re-distributed to households. Second, the effect of tax on rent is negative if tax revenue is not re-distributed back to the households, but is positive if the revenue is re-distributed back. In other words, the tax and transfer scheme discourage housing investment, but encourage housing consumption. Not surprisingly, the tax effect is again stronger when the tax rate is higher. Third, land price is negatively impacted by the tax, whether the tax revenue is re-distributed or not. If the tax revenue is not re-distributed, then land price is more sensitive to the tax than house price. However, when tax revenue is re-distributed, land price is less sensitive to the tax than house price. Evidently, in the latter case the rising housing consumption demand partly offsets the falling housing investment demand, thus the land price is lowered to a lesser extent.

Thus whether tax revenue is re-distributed to households makes a key difference. Intuitively, if tax revenue is not re-distributed, imposing the tax is similar to lowering the income of households, which drives down both consumption demand and investment demand for housing, leading to lower price and rent. However if tax revenue is re-distributed, then poor households benefit relatively more from the tax policy. These households are mainly renters (especially when they are young). As they benefit from the tax policy, their housing consumption demand increases, hence the rental rate of houses.

The middle panel of Table 6 reports the price changes when the property tax is imposed only on the housing for investment purposes. The negative effects on house price, house rent and land price are very similar to the case of universal property tax, but only slightly weaker. In the case of tax revenue re-distribution, the effect on house rent is again positive, slightly stronger than the case of a universal property tax.

The bottom panel reports changes due to land value tax. Not surprisingly, land price is strongly negatively affected. With lower land price, house price is also lowered because land is the most important input in the housing production in our model. Due to the income effect, when the tax revenue is re-distributed, rent rises as in the case of property tax.

**Impact of the Property Tax During the Transition**

Results in the absence of a property tax are studied in [Han et al. (2017)](http://example.com). A key result there is that house price and rent in Beijing in 2014 are roughly consistent with the fundamentals if either: (i) the average income level of home buyers in Beijing is 80% higher than that reported by the National Bureau of Statistics; or (ii) endogenous migration of rich households to Beijing is allowed. In this study we take the case of high average income of home buyers in Beijing, and solve for price and rent prior to the implementation of property tax, then focus on how the variables of interest change when the property tax is implemented.
Non-redistributive Tax

We start with the case of non-redistributive tax. Figure 4 reports the impact on house price, rent, land price and income from land concession (all in terms of 2012 RMB) in the case of the universal property tax. The rate of property tax is assumed to be 1%. Clearly all house price and land concession income are lowered by the tax. Intuitively, property tax lowers the effective return on housing investment, hence the demand for housing investment is reduced. In response, house price is lowered, which reduce land price and hence land concession income is also lowered.

House rent is affected in two ways. First, there is an income effect: with lower return on housing investment households essentially have a tighter budget constraint, which reduces housing consumption demand. Second, there is a substitution effect: with lower return on housing investment households substitute investment for consumption, which increases housing consumption. Since the income effect and substitution effect partly cancel each other, the net effect of the property tax on house rent is only slightly negative. Actually, when tax rate is set at 0.5%, we find that substitution effect dominate income effect when property tax is first implemented, and house price rent initially before it finally falls.

Figure 4: Changes Due to Property Tax (τ = 0.01)

This figure plots the changes of house price, house rent, land price and land concession income (in 2012 RMB) due to a property tax of 1%. The dashed lines represents the paths if the property tax is never implemented.

Of course property tax constitutes another source of government revenue in addition to
land concession, and the total revenue may rise as the result of implementing the property tax. This is shown in Figure 5 where land concession income, property tax revenue and total revenue are normalized to 1 in 2014. The figure shows that total revenue rises immediately after the property tax is implemented, then grows steadily over time. Comparing the paths of land concession income and property tax revenue reveals a critical advantage of the property tax – it rises steadily over time, providing a stable source of revenue for the government. By contrast, land concession income peaks than falls because less and less new land is available.

Figure 5: Revenues in Response to Non-redistributive Property Tax

This figure plots the paths of land concession income, property tax revenue and total revenue. The universal property tax (τ = 0.01) that applied to each home owner is implemented in the end of 2017. All the values in 2015 is normalized to 1.

The top panel of Table 7 reports the percentage changes of house price, house rent, land price, income from land concession and total revenue immediately after the implementation of property tax of 0.0%, 1% and 2%. These are the changes between year t when the new tax policy is implemented, and year t + 1. Given each property tax rate, the prices and land concession income all declines. A higher tax rate is associated with greater declines. On the other hand, total government revenue, i.e., the sum of land concession income and revenue from the property tax, increases with the property tax rate. The changes in price and revenue are quite sensitive to the tax rate.

When the tax rate is 0.5%, the initial response of house rent is a 12.74% increase, as shown in the first row of Table 7. The reason has been discussion in the analysis of Figure 4—substitution effect of the tax dominate income effect when the tax is initially implemented. Of course as households have less assets, later on housing consumption will decline and then house rent also
declines, as is evidenced by the rent level in the BGP shown in Table 6. When the tax rate is higher, we find that income effect dominates and house rent is lowered by the non-redistributive tax.

<table>
<thead>
<tr>
<th>tax rate ( (\tau) )</th>
<th>house price</th>
<th>house rent</th>
<th>land price</th>
<th>land concession income</th>
<th>total revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Universal Property Tax (All Home Owners)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5%</td>
<td>-5.25</td>
<td>12.74</td>
<td>-21.46</td>
<td>-21.63</td>
<td>33.77</td>
</tr>
<tr>
<td>1%</td>
<td>-9.28</td>
<td>-0.22</td>
<td>-27.72</td>
<td>-27.87</td>
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</tr>
<tr>
<td>2%</td>
<td>-15.00</td>
<td>-14.47</td>
<td>-36.10</td>
<td>-36.24</td>
<td>177.43</td>
</tr>
<tr>
<td>Selective Property Tax (Multiple Home Owners)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5%</td>
<td>-5.16</td>
<td>12.42</td>
<td>-21.19</td>
<td>-21.36</td>
<td>34.37</td>
</tr>
<tr>
<td>1%</td>
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<td>-1.94</td>
<td>-27.57</td>
<td>-27.73</td>
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</tr>
<tr>
<td>2%</td>
<td>-11.05</td>
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<td>-31.64</td>
<td>-31.80</td>
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<tr>
<td>Land Value Tax</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5%</td>
<td>0.00</td>
<td>0.00</td>
<td>-25.33</td>
<td>-25.49</td>
<td>17.32</td>
</tr>
<tr>
<td>1%</td>
<td>0.00</td>
<td>0.00</td>
<td>-35.38</td>
<td>-35.53</td>
<td>38.57</td>
</tr>
<tr>
<td>2%</td>
<td>0.00</td>
<td>0.00</td>
<td>-43.09</td>
<td>-43.22</td>
<td>87.29</td>
</tr>
</tbody>
</table>

This table report the percentage changes of house price, house rent, land price and government income (in 2012 RMB) upon the implementation of new tax policies, with tax rate of 0.05%, 1% and 2% respectively. Tax revenue is not redistributed to the households.

The changes in response to a selective property tax is reported in the middle panel of Table 7. Again these are the changes between year \( t \) when the new policy is implemented and year \( t + 1 \). The changes are smaller compared to those caused by the universal property tax given the same tax rate. Overall, when tax revenue is not returned to households, the effects of the selective tax is somewhat similar to universal tax, partly because a large fraction of households owns multiple homes.

The bottom panel of Table 7 shows the effects of the land value tax. The tax effect is similar to that in the BGP – house price and rent are not affected by construction, and land price is lowered significantly. As a result, revenue from land concession is also reduced significantly. Nevertheless, total revenue is increased. That is, the additional government revenue from the land value tax is large enough to cover the loss in land concession income. Comparing the changes of total revenues under different types of taxes, we find that the selective property tax leads to the most increase in total government revenue. Intuitively, the selective property tax causes relative small changes to house price and land price thus land concession income is
reduced to a lesser extent. In addition, a large fraction of households have multiple properties, hence tax revenue is high.

Redistributive Property Tax

If tax revenue is distributed evenly among households, the optimization behavior of households is changed by both the tax and the transfer. Compared with the case of non-redistributive tax, on average households are faced with less constraining budget. Thus the results are quite different.

The top panel of table 8 reports the percentage changes of some key variables due to the implementation of the universal property tax. Notably, rental rate rises rather than falls when the tax rate is 0.5% and 1%. Intuitively, renters benefit from the transfer of tax revenue, so they have more housing consumption demand, driving up rents. When the tax rate is 2%, the income effect of the reduced return on housing investment is dominating, and house rent is reduced by the tax.

<table>
<thead>
<tr>
<th>tax rate (τ)</th>
<th>house price</th>
<th>house rent</th>
<th>land price</th>
<th>land concession income</th>
<th>total revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Universal Property Tax (All Home Owners)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5%</td>
<td>-6.56</td>
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</tr>
<tr>
<td>1%</td>
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<td>2%</td>
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<td>-7.93</td>
<td>-35.70</td>
<td>-35.84</td>
<td>178.11</td>
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<tr>
<td>Selective Property Tax (Multiple Home Owners)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5%</td>
<td>-5.80</td>
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<td>-21.46</td>
<td>-21.63</td>
<td>34.29</td>
</tr>
<tr>
<td>1%</td>
<td>-9.89</td>
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<td>-27.13</td>
<td>-27.29</td>
<td>82.89</td>
</tr>
<tr>
<td>2%</td>
<td>-15.52</td>
<td>-1.96</td>
<td>-35.46</td>
<td>-35.60</td>
<td>177.97</td>
</tr>
<tr>
<td>Land Value Tax</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5%</td>
<td>1.67</td>
<td>12.37</td>
<td>-24.68</td>
<td>-24.84</td>
<td>18.34</td>
</tr>
<tr>
<td>1%</td>
<td>0.81</td>
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<td>-35.27</td>
<td>-35.41</td>
<td>38.81</td>
</tr>
<tr>
<td>2%</td>
<td>0.43</td>
<td>13.25</td>
<td>-49.14</td>
<td>-49.25</td>
<td>67.40</td>
</tr>
</tbody>
</table>

This table report the percentage changes of house price, house rent, land price and government income (in 2012 RMB) upon the implementation of new tax policies, with tax rate of 0.05%, 1% and 2% respectively. Tax revenue is redistributed evenly to the households.

Compared with the results from non-redistributive tax, one can see that the impact of the universal property tax on house price, land price, and land concession income is weaker. This is due to the income effect of the transfer. Total tax revenue is higher under the re-distributive
regime compared to the non-redistributive one, because both housing demand and housing price are relatively higher when tax revenue is transferred back to the households. As a result of a smaller reduction in land concession income, redistributive property tax increases total government revenue to a greater extent than the non-redistributive property tax.

The effects of a selective property tax, as shown in the middle panel, are also different from the universal property tax. It turns out that the selective property tax lowers house price and raises rents more than the universal property tax. In other words, tax effect is stronger with selective property. This is because a selective property tax targets those who are suppliers of rental houses, which reduces supply. In the meantime, demand for rental houses is increased due to the transfer from tax revenue. Consequently, rents rise significantly.

Compared with the results of non-redistributive tax reported in Table 7, total government revenue is higher when tax revenue is redistributed evenly to households.

With redistribution of tax revenue, a land value tax relaxes the budget constraint of households, thus it raises house price and rents in the BGP as well as during the transition. Total government revenue is raised by the tax because additional tax revenue over-compensates the loss in land concession income.

Vacancy

There are considerable debates on what the vacancy rates are in Chinese cities. According to the 2013 wave of China Household Finance Survey (CHFS), the vacancy rate of residential houses in Beijing was 19.5% in 2013. However, there is no official release of the vacancy rate, and the CHFS vacancy rate is questioned by the National Bureau of Statistics. Consequently there is no consensus regarding what the vacancy rate is in Beijing as well as other cities in China.

In this section, we evaluate the potential impact of various tax and transfer schemes on vacancy rate. To endogenously generate vacancy rate in our structural model, some market friction needs to be introduced. Theoretical research on vacancy rate is limited. Two influential papers that use a search model to endogenously generate vacancy rate of residential housing are Wheaton (1990) and Diaz and Jerez (2013). Both papers focus on owner-occupied houses. In


26At a press conference held by the NBS, Ma Jiantang, director of the NBS, stated: “It is almost impossible to determine what kind of situation qualifies as vacant housing. For example, is a vacant home one that nobody lives in, or one that has been empty for over half a year? It is not only a question of where; it is also a question of time, and thus difficult to come up with a standard.” See http://www.eeo.com.cn/ens/2010/0903/179867.shtml
our paper, we focus on the vacancy of rental units. For simplicity and tractability, we introduce a cost in the rental market which captures frictions due to search and information asymmetry, transaction and maintenance costs as well as moral hazard arising from damage due to improper use by tenants. When the renting cost is sufficiently high, owners of residential houses leave the houses vacant.

Specifically, in the beginning of each period, a home owner receives a random renting cost which is drawn from a uniform distribution between zero and \( \bar{\phi}_t \). If the cost is lower than the rent in that period, then the owner lists the house on the market which becomes part of the supply in the rental market. On the other hand, if the cost is higher than the rent, the house is left vacant in that period. This modeling strategy captures not only the intuition that the vacancy rate should decrease as rent increases, but also the intuition that a high vacancy rate is consistent with lower housing supply. The latter is achieved endogenously in our model because a high vacancy rate is associated with a low return on housing investment resulting in less housing investment demand. Through the general equilibrium effect, housing supply will also be reduced.

We modify the baseline model so that a time-invariant natural vacancy rate exists when the economy operates in the BGP. To construct the time-invariant natural vacancy rate, it is necessary for the upper bound of the uniform distribution of the renting cost to increase at the same pace as the rent in the BGP. In the Proposition we have shown that the growth factor of rent in the BGP is \( (G_Y / G_L)^\theta \), thus we assume \( \bar{\phi}_t \) also grows at a factor of \( (G_Y / G_L)^\theta \) in the BGP.

The value of \( \bar{\phi}_t \) in the BGP is calibrated based on the model without any property tax or land value tax. We target a natural vacancy rate of 5% in the BGP which is close to the average rental vacancy rate in the US cities. Since the renting cost is evenly distributed between zero and \( \bar{\phi}_t \) in the BGP a \( \bar{\phi}_t \) that is 5% higher than the rent generates a vacancy of 5%. To pin down the values of \( \bar{\phi}_t \) during the transition of the economy, we assume vacancy rate is 10% on average between 2018-2028, then it declines to 5% when the economy enters the BGP, implying that \( \bar{\phi}_t \) is 10% higher than the rent initially, then its value relative to the rent declines in a linear fashion as well.

Using the \( \bar{\phi}_t \), we re-computing house price, rent and vacancy rate when the economy enters the BGP under various tax and transfer regimes. In computing the equilibrium rent path, we search for the rents that equates total housing consumption demand with the total housing supply posted on the rental market.

Results for the economy in the GBP are reported in Table 9. Note that house price and rent are different from the baseline model where the vacancy rate is always zero. However, a comparison with Table 6 shows that house prices and rents in the model with vacancy are very similar to those in the baseline, and their changes due to tax and transfer policies are almost
The main interest lies in the vacancy rate. The model predicts that property tax without transfer raises the natural vacancy rate in the BGP, but property tax with transfer lowers the natural vacancy rate. This is caused by changes in house rent. If a policy raises house rent, then more home owners will find that the rent exceeds renting cost and post their rental properties on the market. This happens when the tax revenue is transferred to households. The effect of tax and transfer on vacancy is stronger when only investment housing is taxed (i.e. the case of

| tax rate (\(\tau\)) | Non-redistributive | Redistributive | |
|----------------------|---------------------|----------------|
|                      | house price | house rent | vacancy rate (%) | house price | house rent | vacancy rate (%) |
| 0%                   | 143.47     | 4.61      | 5.00              | 143.47     | 4.61      | 5.00              |
| Property Tax (All Owners) |
| 0.5%                 | 130.30     | 4.51      | 6.29              | 134.62     | 4.89      | 0.24              |
|                      | -9.2       | -2.2      | 25.79             | -6.2       | 6.0       | -95.26            |
| 1%                   | 119.43     | 4.44      | 7.60              | 125.98     | 5.13      | 0.18              |
|                      | -16.8      | -3.7      | 52.08             | -12.2      | 11.3      | -96.42            |
| 2%                   | 102.73     | 4.37      | 9.24              | 116.79     | 5.19      | 0.16              |
|                      | -28.4      | -5.3      | 84.74             | -18.6      | 12.6      | -96.71            |
| Property Tax (Multiple House Owners) |
| 0.5%                 | 130.38     | 4.51      | 6.23              | 134.60     | 4.89      | 0.24              |
|                      | -9.1       | -2.2      | 24.58             | -6.2       | 5.9       | -95.26            |
| 1%                   | 119.42     | 4.45      | 7.60              | 126.13     | 5.17      | 0.17              |
|                      | -16.8      | -3.6      | 51.95             | -12.1      | 12.1      | -96.55            |
| 2%                   | 102.33     | 4.35      | 9.72              | 117.29     | 5.29      | 0.14              |
|                      | -28.7      | -5.8      | 94.32             | -18.2      | 14.6      | -97.24            |
| Land Value Tax |
| 0.5%                 | 143.47     | 4.61      | 5.00              | 147.90     | 4.75      | 1.23              |
|                      | 0         | 0         | 0                 | 3.1        | 3.1       | -75.32            |
| 1%                   | 143.47     | 4.61      | 5.00              | 149.23     | 4.93      | 0.23              |
|                      | 0         | 0         | 0                 | 4.0        | 6.8       | -95.45            |
| 2%                   | 143.47     | 4.61      | 5.00              | 147.66     | 5.25      | 0.15              |
|                      | 0         | 0         | 0                 | 2.9        | 13.9      | -97.05            |

This table reports house price, house rent, and vacancy rate when the economy enters the BGP (in 2012 RMB) under various tax and transfer regimes. The italic numbers are percentage changes in prices relative to the zero-tax regime.

The main interest lies in the vacancy rate. The model predicts that property tax without transfer raises the natural vacancy rate in the BGP, but property tax with transfer lowers the natural vacancy rate. This is caused by changes in house rent. If a policy raises house rent, then more home owners will find that the rent exceeds renting cost and post their rental properties on the market. This happens when the tax revenue is transferred to households. The effect of tax and transfer on vacancy is stronger when only investment housing is taxed (i.e. the case of
taxing owners of multiple homes) because this type of tax raises rent more than the universal property tax.

The bottom panel of Table 9 reports the effects of land value tax. Since this type of tax raises house rent, it effectively lowers vacancy rate. The model predicts very strong effects: a 0.5% land value tax lowers vacancy rate from 5% to 1.23%, amounting to a decrease of 75.32%.

Table 10 reports tax effect on vacancy between 2018-2028, the first 10 years after the implementation of tax policy. Consistent with the tax effect in the BGP, the non-redistributive tax (i.e. taxation without transfer) leads to higher vacancy rate, but the redistributive tax effectively lowers vacancy rate. With higher tax rate, policy effect is unequivocally stronger. The tax effect is stronger when the property tax is selective, i.e. only owners of multiple homes are taxed. A selective property tax of 2% combined with transfer causes a 64.4% reduction in vacancy rate. Given that this tax scheme has been shown to raise total government revenue despite a 31.8% decline in land concession income, as shown in Table 8, it is a scheme to be recommended.

Table 10: Vacancy Rate During the Transition

<table>
<thead>
<tr>
<th>Tax Rate</th>
<th>Non-redistributive</th>
<th>Redistributive</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>vacancy(%) changes(%)</td>
<td>vacancy(%) changes(%)</td>
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This table reports the average vacancy rates within 10 years after the implementation of various tax regimes (between 2018-2028). The italic numbers are percentage changes relative to the zero-tax regime which has a vacancy rate of 10% between 2018-2018.
Conclusion

Within a dynamic general equilibrium framework, this paper predicts the quantitative effects of three types of tax policies: a universal property tax applied to all the home owners, a selective property tax applied to home owners with multiple houses, and a land value tax. Quantitative effects are reported regarding house price, house rent, land price, vacancy rate, government income from land concessions, revenue from taxation and total government revenue.

In the scenario where tax revenue is not redistributed to households, a property tax lowers house price, house rent and land price, but it increases vacancy rate. Land concession income is also lowered due to the reduced land price. But total revenue is increased due to the larger additional revenue from the property tax.

In the scenario where tax revenue is transferred evenly to households, a property tax lowers house price and land price, but increases house rent. It also effectively lowers the vacancy rate. The tax effect is weaker when the property tax is universal. In other words, a selective property tax reduces house price, increases house rent and lowers the vacancy rate to a larger extent than a universal tax. This is because a selective tax discourages the ownership of multiple houses, which essentially lowers housing supply in the rental market. Overall, total government revenue is very similar with a selective property tax compared with a universal property tax.

A land value tax reduces land price and land concession income. When tax revenue is not redistributed, the reduction in land price is so significant that total government revenue, the sum of land concession income and land value tax revenue, is reduced. When tax revenue is redistributed evenly to households, land price is reduced to a much lesser extent, and total government revenue is increased. By construction, the non-redistributive land value tax has no impact on house price and house rent. But the redistributive land value tax increases house price and rent due to the income effect – households are faced with a less constraining budget due to government transfer.

Overall, the redistributive universal property tax that applies to owners of multiple homes is more favourable because it impacts house price and rent to a less extent but increases total government revenue effectively. It is also the most effective in reducing vacancy rate. When the tax rate is 1%, the implementation of a selective property tax (with transfer) reduces house price and land price by 9.89%, and 27.13% respectively. It increases house rent by 3.94%, and increases total revenue by 82.89%. It also lowers the natural vacancy rate by 96.55% at the BGP, and lowers the vacancy rate by 64.4% during the first 10 years after implementation. In addition, total tax revenue rises steadily with income and house price over time, amounting to
a sustainable source of revenue for local governments in China. This appears to be the most effective tax and redistribution set of policies for China going forward.

**Proof of the Proposition**

Given the exogenous growth factors of the fundamentals such as income, population and land supply, we will show that the growth factors of prices \( \{G_p, G_r, G_q\} \) and the growth factors of the choice variables of the firm and households, as proposed in the Proposition, satisfy the general equilibrium conditions. Specifically they are consistent with: (i) the optimization problem of the firm; (ii) the optimization problems of households; (iii) the market clearing conditions.

We prove the proposition by induction. Let \( K^*_t, L^*_t, H^*_t \) denote the firm’s optimal decisions and \( b^*_{a,t}, c^*_{a,t}, s^*_{a,t}, h^*_{a,t} \) denote a household’s optimal decisions in period \( t \). Further let \( p_t, r_t, q_t \) be the market clearing house price, rent and land price. By definition, these choice variables and prices satisfy the general equilibrium conditions in period \( t \). We will show that, using the proposed growth factors, \( K_{t+1} = K^*G_K, L_{t+1} = L^*G_L, H_{t+1} = H^*G_H, b_{a,t+1} = b^*_{a,t}G_b, c_{a,t+1} = c^*_{a,t}G_c, h_{a,t+1} = h^*_{a,t}G_h, s_{a,t+1} = s^*_{a,t}G_s, p_{t+1} = p_tG_p, r_{t+1} = r_tG_r \) and \( q_{t+1} = q_tG_q \) will also satisfy the three general equilibrium conditions in period \( t + 1 \). In particular, we show that the decisions of households of age \( a \) at time \( t + 1 \) are the same as the earlier cohort (those of age \( a \) at time \( t \)) up to the scale factors as proposed in the Proposition, which guarantees that the average consumption and investment will grow at the proposed factors because the age distribution of households is time-invariant in the BGP.

**Firm’s Optimization Problem**  First we show that the growth factors are consistent with the firm’s flow of funds equation:

\[
p_t(H_t^* - H_{t-1}) = K_t^* - (1 - \delta)K_{t-1} + q_t(L_t^* - L_{t-1}).
\]

Assuming the equation above holds in period \( t \), we want to show that its counterpart in period \( t + 1 \) holds as well. We multiply both sides of the above equation with \( G_Y \) and apply the proposed growth factors. The left side of the equation is

\[
p_t(H_t^* - H_{t-1})G_Y = p_t(H_t^* - H_{t-1}) \left( \frac{G_Y}{G_L} \right)^\theta G_Y^{1-\theta}G_L^\theta
\]

\[
= p_t(H_t^* - H_{t-1})G_pG_H
\]

\[
= p_tG_p(H_t^*G_H - H_{t-1}G_H)
\]

\[
= p_{t+1}(H_{t+1}^* - H_t).
\]
The right side is

\[
[K^*_t - (1 - \delta)K_{t-1} + q_t(L^*_t - L_{t-1})]G_Y = K^*_tG_Y - (1 - \delta)K_{t-1}G_Y + q_t(L^*_t - L_{t-1})G_Y
\]

\[
= K^*_tG_K - (1 - \delta)K_{t-1}G_K + q_t(L^*_t - L_{t-1})\frac{G_Y}{G_L}G_L
\]

\[
= K^*_{t+1} - (1 - \delta)K_{t-1}G_K + q_t(L^*_{t+1} - L_{t-1})G_K
\]

\[
= K^*_{t+1} - (1 - \delta)K_{t-1}G_K + q_{t+1}(L^*_{t+1} - L_{t-1}G_L).
\]

Therefore, given state variables \( \{K_t, L_t\} = \{K_{t-1}G_K, L_{t-1}G_L\} \) we have

\[
p_{t+1}(H^*_{t+1} - H_t) = K^*_{t+1} - (1 - \delta)K_t + q_t(L^*_{t+1} - L_t)
\]

That is, the flow of funds condition in period \( t + 1 \) can be derived from that in period \( t \) and the growth factors in the Proposition.

Next, we show the proposed growth factors are consistent with the firm’s first-order condition with respect to \( K \) which is

\[
Z(1 - \theta)\tilde{r}_t\left(\frac{K^*_t}{L_t}\right)^{-\theta} = 1 - \frac{1 - \delta}{R_t}
\]  \hspace{1cm} (38)

First of all, notice that in the BGP, \( G_p = G_r \), therefore

\[
R_t = \frac{p_t + r_t}{p_{t-1}} = \frac{G_p(p_t + r_t)}{G_p p_{t-1}} = \frac{p_{t+1} + r_{t+1}}{p_t} = R_{t+1}.
\]

Thus housing return is a constant in the BGP, and \( \tilde{r}_t = r_{t+1}/R_{t+1} \) grows at the same factor as \( r_{t+1} \). Given (38), to prove the same first-order condition with respect to \( K \) holds in period \( t + 1 \), it suffices to show \( \tilde{r}_t(K^*_t/L^*_t)^{-\theta} = \tilde{r}_{t+1}(K^*_{t+1}/L^*_{t+1})^{-\theta} \), which is straightforward using the proposed growth factors. Thus we omit the details and conclude that

\[
Z(1 - \theta)\tilde{r}_{t+1}\left(\frac{K^*_{t+1}}{L^*_{t+1}}\right)^{-\theta} = 1 - \frac{1 - \delta}{R_{t+1}}
\]

Third, we show the proposed growth factors are consistent with the firm’s first-order condition with respect to \( L \) which is

\[
Z\theta\tilde{r}_t\left(\frac{K^*_t}{L^*_t}\right)^{1-\theta} = q_t - \frac{q_{t+1}}{R_t}
\]

We need to show that

\[
Z\theta\tilde{r}_{t+1}\left(\frac{K^*_tG_K}{L^*_{t}G_L}\right)^{1-\theta} = q_{t+1} - \frac{q_{t+2}}{R_{t+1}}
\]
Starting from the left-side of the equation above and substituting the proposed growth factors in the Proposition, we have

\[
Z \theta \tilde{r}_{t+1} \left( \frac{K^*_t G_K}{L^*_t G_L} \right)^{1-\theta} = Z \theta \tilde{r}_t G_r \left( \frac{K^*_t}{L^*_t} \right)^{1-\theta} \left( \frac{G_K}{G_L} \right)^{1-\theta}
\]

\[
= Z \theta \tilde{r}_t \left( \frac{G_L}{G_Y} \right)^\theta \left( \frac{K^*_t}{L^*_t} \right)^{1-\theta} \left( \frac{G_Y}{G_L} \right)^{1-\theta}
\]

\[
= Z \theta \tilde{r}_t \left( \frac{K^*_t}{L^*_t} \right)^{1-\theta} \left( \frac{G_Y}{G_L} \right)^{1-\theta}
\]

\[
= Z \theta \tilde{r}_t \left( \frac{K^*_t}{L^*_t} \right)^{1-\theta} G_Y \frac{G_L}{G_L}
\]

\[
= \left( \frac{q_t}{R_t} - \frac{q_{t+1}}{R_t} \right) G_q
\]

\[
= q_{t+1} - \frac{q_{t+2}}{R_{t+1}}
\]

where the last equality holds because housing equity return is constant in the BGP.

**Household’s Optimization Problems** Using the same strategy as with the firm’s flow of fund equation, it is straightforward to show that the growth factors of \( b_{a,t}, c_{a,t}, s_{a,t}, h_{a,t} \) are consistent with the households’ budget constraint. The intra-temporal optimal allocation in household’s problem is governed by equation (25). It is also straightforward to show that the proposed growth factors are consistent with this equation. For brevity, we omit the algebraic details which are available upon request.

To complete the proof that the proposed growth factors are consistent with household’s optimization problems, we show that the functional equations that define \( V_{own}, V_{rent}, W_{own} \) and \( W_{rent} \) are all re-scalable so that if \( b_{a,t}^*, c_{a,t}^*, h_{a,t}^*, s_{a,t}^* \) solve the optimization problems in period \( t \), then \( b_{a,t+1} = b_{a,t}^*G_b, c_{a,t+1} = c_{a,t}^*G_c, h_{a,t+1} = h_{a,t}^*G_h, s_{a,t+1} = s_{a,t}^*G_s \) will solve the optimization problems of households with the same age \( a \) in period \( t + 1 \).

First, using the growth factors proposed the Proposition, we show that both the bequest value and the utility function can be re-scaled by \( (G_Y^{1-\theta}G_L^{\theta}/G_N)^{1-\gamma} \). From equation (17), in period \( t + 1 \) we have

\[
V_b(s_{a,t+1}) = B \left[ (1 - \omega)^{1-\omega} (\omega)^\omega \right]^{1-\gamma} \left( \frac{1}{r_{t+1}} \right)^{\omega(1-\gamma)} \frac{(p_{t+1}s_{a,t+1})^{1-\gamma}}{1-\gamma}
\]

\[
= B \left[ (1 - \omega)^{1-\omega} (\omega)^\omega \right]^{1-\gamma} \left( \frac{1}{r_t} \right)^{\omega(1-\gamma)} \frac{(p_t s_{a,t})^{1-\gamma}}{1-\gamma} \left( \frac{G_p G_s}{G_r^{\omega}} \right)^{1-\gamma}
\]

\[
= V_b(s_{a,t}) \left( \frac{G_p G_s}{G_r^{\omega}} \right)^{1-\gamma}
\]

\[
= V_b(s_{a,t}) \left( \frac{G_Y^{1-\theta}G_L^{\theta}/G_N}{G_N} \right)^{1-\gamma}
\]

(39)
where we used the proposed growth factors for the last equality. For households whose bequest is in the risk-free asset, it is straightforward to show that $V_b(b_{a,t+1}) = V_b(b_{a,t}) \left( \frac{G_Y^{1-\theta} G_L^\theta}{G_N} \right)^{1-\gamma}$.

Similarly, for the utility function we have

$$u(c_{a,t+1}, h_{a,t+1}) = u(c_{a,t}, h_{a,t}) \left( \frac{G_Y^{1-\theta} G_L^\theta}{G_N} \right)^{1-\gamma}. \quad (40)$$

Finally, using equation (39) and (40), we can show via backward inductions that the household’s value functions at any age can be re-scaled by $\left( \frac{G_Y^{1-\theta} G_L^\theta}{G_N^{1-\gamma}} \right)^{1-\gamma}$. This property, combined with the fact that the household’s budget constraints are consistent with the proposed growth factors, implies that the functional equations are re-scaldable.  

**Market Clearing** We need to show that the proposed growth factors are consistent with the clearing of land market, housing consumption market and equity market. For the land market, it is sufficient to show that land demand grows at the same factor as the exogenous land supply $G_L$. The firm chooses an optimal land input in housing production given the land price, according to the first-order condition given by equation (7), which implies that

$$G_r \left( \frac{G_K}{G_L^d} \right)^{1-\theta} = G_q,$$

where $G_L^d$ is the growth factor of land demand in the BGP. It follows that

$$G_L^d = \left( \frac{G_r}{G_q} \right)^{1/(1-\theta)} G_K. \quad (41)$$

Substituting the growth factors $G_r$, $G_q$, and $G_K$ proposed in the Proposition, we get $G_L^d = G_L$, hence the growth factors satisfy the land market clearing condition.

Next we show that the growth factors satisfy the housing consumption market clearing condition. Aggregate housing supply grows at a factor of $G_H$, while households’ housing consumption demand grows at a factor of $G_h$. It is enough to show that $G_H/G_N = G_h$. By the housing production function (1), $G_H = G_K^{1-\theta} G_L^\theta$. Given $G_Y = G_K$ (see point 1 of the Proposition), we have

$$\frac{G_H}{G_N} = \frac{G_Y^{1-\theta} G_L^\theta}{G_N} = \left( \frac{G_Y}{G_N} \right)^{1-\theta} \left( \frac{G_L}{G_N} \right)^\theta = G_h. \quad (42)$$

Therefore, these growth factors $G_H$ and $G_h$ as given in points 2-3 of the Proposition satisfy the housing consumption market clearing condition. The same argument applies to the home equity market clearing condition with $G_h$ replaced by $G_s$.

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27For technical reason, we assume that in the BGP, the minimum housing size grows at a constant factor of $(G_Y/G_N)^{1-\theta}(G_L/G_N)^\theta$, which is the same as the growth factor $G_s$ of housing investment demand and the growth factor $G_h$ of consumption demand. This assumption guarantees that the minimum housing size constraint is equally binding in each period in the BGP and helps to preserve the scalability of households’ optimization problems.
FAR, Price-income Ratio and Price-rent ratio  We have shown that growth factors in points 1-8 of the Proposition satisfy all the general equilibrium conditions. Using these growth factors, we have $FAR_{t+1}/FAR_t = {H_{t+1}/L_{t+1} \over H_t/L_t} = G_H/G_L = \frac{G_Y/G_L}{\theta Y/G_L} = (G_Y/G_L)^{1-\theta}$, which is point 9 of the Proposition. Also it is straightforward to show that both price-income ratio and price-rent ratio are time-invariant. Hence point 10 of the Proposition is true.
References


