Most jurisdictions require residential assessments to be proportional to market value, but in practice assessment ratios—assessed value divided by sale price—are often lower for high-priced than low-priced properties. This tendency for assessment ratios to fall as sales prices rise is termed **regressivity**, because it means that property taxes are a higher percentage of property value for lower-priced properties. Regressive assessments have been identified in many jurisdictions and times (such as Cornia and Slade 2005; McMillen and Weber 2008; and Plummer 2010).

Assessment regressivity is an important issue because it has the potential to undermine support for a property tax system. Consider a simple system in which taxes are 1 percent of a home’s assessed value, with no exemptions or deductions. For example, a $100,000 home should have a $1,000 tax bill, and a $1 million home a $10,000 tax bill. However, it is not uncommon to find that a $1 million home is actually assessed at $800,000 or $900,000, resulting in effective tax rates of 0.8 or 0.9 percent rather than the statutory 1 percent.

Having lower-than-prescribed assessment rates for some high-priced properties may result in greater variability in assessments within price groups. One owner of a high-priced home may accept a $1 million assessment as an accurate measure of market value, while another may appeal and win a lower assessment. Different tax bills for identical properties can cause taxpayer resistance and resentment.

*Daniel P. McMillen*
TABLE 1
Traditional Assessment Performance Measures

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>9.4%</td>
<td>29.8%</td>
</tr>
<tr>
<td>Median</td>
<td>9.2%</td>
<td>29.9%</td>
</tr>
<tr>
<td>Value-Weighted Mean</td>
<td>9.0%</td>
<td>29.2%</td>
</tr>
<tr>
<td>Price-Related Differential</td>
<td>1.047</td>
<td>1.021</td>
</tr>
<tr>
<td>Coefficient of Dispersion</td>
<td>18.279</td>
<td>8.702</td>
</tr>
</tbody>
</table>

Source: Author calculations based on data from the Illinois Department of Revenue.

The Assessment Process in Illinois

I have analyzed data from two counties in the Chicago metropolitan area that provide quite different perspectives on assessment regressivity. In suburban DuPage County, assessment ratios decline uniformly with sales prices and there is no marked difference in the degree of variability in assessments across the range of sales prices. In the City of Chicago, which is part of Cook County, the degree of variability in assessment ratios is greater than the degree of regressivity. Notably, assessment ratios in Chicago are highly variable at low and very high sales prices, while not varying greatly with mid-range sales prices.

Illinois has a simple flat-rate property tax, but the homestead exemption produces a degree of progressivity. This exemption is generally a flat amount that does not vary by price, although Cook County has an “alternative general homestead exemption” that can make the exemption higher in areas with rapid price appreciation. The basic homestead exemption is designed to produce much lower effective tax rates for low-priced properties—where the exemption is often high relative to market value.

Assessment practices in DuPage County are similar to those in all but one of the 102 counties in Illinois, where properties are assessed on a four-year cycle at 33 percent of market value. In DuPage County, properties were most recently assessed in 2007 and new assessments will be established in 2011. Cook County alone has a classified system with varying statutory assessment rates. Prior to 2009, the statutory rates were 16 percent for residential properties, 38 percent for commercial, and 36 percent for industrial, although actual assessment rates were much lower. In 2009, the statutory rates were “recalibrated” to 10 percent for residential and 25 percent for commercial and industrial properties. Cook County assesses its properties on a rotating, three-year cycle. The City of Chicago was last reassessed in 2009, and all city properties will be reassessed again in 2012. Properties in the north suburban part of Cook County were reassessed in 2010, and south suburban properties will be reassessed in 2011.

Traditional Measures of Regressivity

The importance of assessment regressivity has led the International Association of Assessment Officers (IAAO 2007) to recommend that an analysis of regressivity be included as part of any study of assessment accuracy. One common procedure recommended by the IAAO to evaluate assessment regressivity is a descriptive statistic, the price-related differential (PRD), which is the ratio of the simple mean assessment ratio to a comparable statistic that places more weight on higher-priced properties. Typically this ratio is greater than one, which implies that higher-priced properties have lower average assessment ratios than lower-priced homes.

Table 1 presents traditional IAAO measures of residential assessment performance for the most recent reassessment year for which I have data—2006 in Chicago and 1999 in DuPage County. The data on sales prices and assessed values come from the Illinois Department of Revenue, which is responsible for monitoring assessment performance for all counties in the state. I focus on Chicago rather than all of Cook County to keep the sample size more manageable, to focus on a single assessment year, and to avoid combining the county’s three assessment districts.

Chicago’s average assessment rate (mean) of 9.4 percent differs significantly from the statutory value of 16 percent. In DuPage County, the average assessment rate of 29.8 percent is much closer to the statutory 33 percent rate, and it would likely be even closer if the timing of the sales prices and assessment origination dates were closer. The value-weighted mean is calculated by weighting each observation by its sale price. The finding that the value-weighted mean is less than the arithmetic mean implies that higher-priced properties tend to have lower than average assessment ratios in both counties.

The price-related differential (PRD), which is the ratio of the value-weighted mean to the arithmetic mean, formalizes this measure. IAAO stan-
standards call for the PRD to be no higher than 1.03; by this standard, DuPage County’s degree of regressivity is acceptable while Chicago’s is not. The coefficient of dispersion (COD) is the traditional measure of assessment variability. By IAAO standards for residential properties, the COD should not exceed 15. Again, Chicago’s COD indicates excessive variability while DuPage County’s degree of variability is within IAAO’s acceptable range.

**Statistical Analysis of Regressivity**

A second IAAO-recommended procedure to measure regressivity is a statistical regression of a sample of assessment ratios on sales prices, which typically produces a negative coefficient for the price variable, i.e., a downward sloping line. This type of analysis provides estimates of the conditional expectation of the assessment ratio for any given sale price. Although several approaches exist in the literature, the basic idea is to estimate a function that produces a simple relationship between sales prices and assessment ratios. If the function implies that assessment ratios decline with sales prices, the assessment pattern is said to be regressive.

Figure 1 shows the estimated functions when assessment ratios are regressed on sales prices using data from Chicago and DuPage County. The straight lines are simple linear regressions. The curved lines are a nonlinear estimation procedure—a locally weighted regression technique that estimates a series of models at various target values, placing more weight on values closer to the target points. For example, to estimate a regression with a target point of $100,000, one might use only observations with sales prices between $75,000 and $125,000, with more weight placed on sales prices closer to $100,000.

The linear and locally weighted regression estimates are much more discrepant for Chicago’s data set than for DuPage County’s. While both approaches indicate that assessment ratios fall with sales prices, the nonlinear procedure indicates that expected assessment ratios are extremely high in Chicago at very low sales prices—but still below the statutory rate of 16 percent.

The regression lines imply precise relationships, but they do not address differences in the degree of variability at different sales prices. It may be that both unusually high and unusually low prices are simply hard to assess accurately. If so, assessment ratios could have high variances at both low and high sales prices while being tightly centered.
Quantile Regressions Using Simulated Data

Another statistical procedure, quantile regression, provides much more information on the relationship between assessment ratios and sales prices by showing how the full distribution of ratios varies by price. The easiest way to understand quantile regression is to imagine two data sets, A and B, where both have 10,000 observations. Each observation represents a sale price and assessment ratio pair, but sales prices are constrained to integers between 1 and 10 (figure 2).

In constructing data set A, a sale price is assigned, and then an assessment ratio is drawn from a normal distribution with a mean (and median) of 0.33 (the statutory rate in DuPage County). Data set A then matches the assumptions of a classical regression model, where the variance of the assessment ratios is constant across all values of sales prices. In constructing data set B, however, the variance of the assigned assessment ratio is higher for lower sale price levels, but the mean is constant and equals 0.33 at each price.

In both data sets the mean is equivalent to the estimated linear regressions in this case, indicating no relationship between sale price and assessment ratio. If these regressions were estimated using real data, they would be interpreted as indicating that assessment ratios are proportional to sales prices, i.e., assessments are neither regressive nor progressive. Despite this finding, figure 2 clearly shows that in data set B assessments converge on the statutory 33 percent rate at high sales prices, whereas homes with low sales prices run the risk of having extremely high assessment rates.

Quantile regression estimates reveal the differences between data sets A and B in the degree of assessment ratio variability, and this approach can be estimated at any target value of the assessment ratio distribution. For example, since the 10 percent and 90 percent quantile lines are converging as sales prices increase, the quantile regression reveals what standard regression procedures do not—low sales prices have highly variable assessments and high sales prices have more precise assessments.
**Quantile Regressions for the City of Chicago and DuPage County**

In practice, linear regression, locally weighted regression, and a linear version of quantile regression all proved too restrictive to represent accurately the relationship between assessment ratios and sales prices in Chicago and DuPage County, especially for extremely low and extremely high sales prices. Instead, a nonlinear version of quantile regression provides the most accurate representation of the underlying relationship.

Figure 3 shows the results of nonlinear versions of the quantile regressions, which can be estimated at a series of target points, with more weight given to observations that are near the targets. From bottom to top, the graphs show the estimated 10, 25, 50, 75, and 90 percent quantile regression lines.

Chicago’s results suggest that assessment ratios are relatively high at all quantiles for quite low prices, but the high variability is evident in the large spread between the 10 and 90 percent quantile lines. However, as the sale price increases from about $250,000 to nearly $800,000, the regression lines are close to horizontal. The variability is also low in this range. The quantile lines begin to have a downward slope again for prices above $800,000, with a moderate increase in the variance. Thus, the Chicago results suggest that the standard analysis of regressivity is misleading in that most of the regressivity is concentrated at low sales prices where the variance is also quite high.

In contrast, DuPage County has relatively high assessment ratios and lower variances in the $100,000–$200,000 range of prices where most sales took place in 1999. Assessment ratios decline with sale price for all prices beyond about $100,000, while the variance is increasing. The pattern of results for DuPage County is closer to what is implicitly assumed in a standard regression analysis of assessment regressivity.

**Assessment Ratios Distributions at Alternative Sales Prices**

An alternative to quantile regression is to examine the actual distribution of assessment ratios at a variety of different target values for sales prices to see how assessment ratios vary at given sales prices. Since most of the interesting patterns occur at low sales prices, figure 4 shows estimated conditional density functions for prices ranging from $50,000 to
Table 2: Representative Homeowner’s Tax Bill in Cook County

<table>
<thead>
<tr>
<th>Estimated Market Value</th>
<th>Assessment Level (10 percent)</th>
<th>Proposed Assessed Valuation</th>
<th>2006 State Equalizer (multiplier)</th>
<th>Equalized Assessed Value (EAV)</th>
<th>Homeowner Exemption</th>
<th>Adjusted Equalized Assessed Value (AEAV)</th>
<th>Sample Tax Rate</th>
<th>Estimated Tax Bill</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100,000</td>
<td>$10,000</td>
<td>$27,000</td>
<td>− $5,500</td>
<td>$21,500</td>
<td>$2,150</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X .10</td>
<td></td>
<td>X 2.7</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Source: Author calculations.

shows what would be predicted by a classic regression analysis of a regressive assessment system.

Implications for Property Taxes

Assessment regressivity has important implications for individual tax bills, as exemplified in a simplified analysis of residential taxes in Cook County. Though not a literal representation of the county’s tax system, the analysis is a close approximation. The starting point for Table 2 is the estimated market value, which we assume to be accurate. Although the statutory assessment rate in Cook County was 16 percent prior to 2009, I use an assessment rate of 10 percent because it is closer to the actual rate and it matches the recent recalibration. Thus, the proposed assessed valuation for the property is $10,000.

However, Illinois also requires that assessments across the state must average 33 percent of market value. If assessments average less than 33 percent—as is mathematically a near certainty under Cook County’s classification system—the Department of Revenue calculates an equalization factor by which all assessments are multiplied. Using a representative value of 2.7 for the multiplier in Table 2, the $10,000 assessment turns into an adjusted equalized assessment value of $27,000. Finally, the standard homestead exemption of $5,500 (again, a representative value) is subtracted to produce the base for the homeowner’s property tax bill. Thus,
the sample tax rate of 10 percent and the adjusted equalized assessed value of $21,500 produce a tax bill of $2,150.

Table 3 compares house values and property tax rates under the assumption that assessments are regressive and are more variable for $100,000 houses than for $500,000 houses. Due to the homestead exemption, the property tax is somewhat progressive even when assessments are proportional to market value. Thus, a $100,000 house that is accurately assessed at 10 percent of market value ($10,000) ends up with a tax bill of $2,150 or an effective tax rate of 2.15 percent, while a $500,000 house that is assessed correctly at $50,000 has a tax bill of $12,950, or 2.59 percent of market value.

But, suppose that assessment rates for $100,000 homes actually range from 9 to 14 percent, while the range for $500,000 homes is only 8 to 12 percent. In this case, the progressivity of the homestead exemption can be reversed completely. Owners of low-priced homes who are “unfortunate” in receiving high assessments end up with effective tax rates of 3.23 percent, which is much higher than the average 10 percent value for owners of $500,000 homes, and is even higher than the 3.13 percent tax rate paid by owners of high-priced homes assessed at 12 percent.

Moreover, actual tax payments vary significantly for otherwise identical homes—from $1,800 to $3,230 for $100,000 houses and from $10,250 to $15,650 for $500,000 homes. In other words, a homeowner may receive a tax bill that is nearly 80 percent higher than the neighboring house even if both have a market value of $100,000.

Conclusion
Because assessment accuracy is the key to an equitable property tax, statistical measures of regressivity are essential tools for evaluating property evaluation systems. Standard measures of regressivity can present an incomplete or even misleading picture of the range of assessment ratios in a jurisdiction. Newer analytic tools such as quantile regression can improve our understanding of the distribution of tax burdens and in this way help improve assessment equity.

NOTE: The statistical tools used in this article are included in a contributed extension package for the statistical program R. The package (aratio) is designed to be accessible to people who have limited knowledge of the R program but are familiar with other statistical software packages. Both R and aratio can be downloaded at no charge from www.r-project.org.